TFY4240

Solution problemset 4 Autumn 2014

NTNU

Institutt for fysikk

Problem 1.

a) In the lectures the electric dipole was placed at the center of the coordinate system and one obtained

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{p}\cdot\hat{\boldsymbol{r}}}{r^2}, \tag{1}$$

for the scalar electric potential. In a coordinate system not located at the center of the dipole, $\boldsymbol{r}$ therefore has to be replaced by the distance from the dipole to the observation point. If we denote this distance by $\boldsymbol{R}$ one thus obtains

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{p}\cdot\hat{\boldsymbol{R}}}{R^2}, \qquad \boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}', \tag{2}$$

which is valid in any coordinate system. Formally the above expression is obtained by a change of spacial variable from the "centered" to the "non-centered" coordinate system.

b) The electric field is obtained in the standard way from

$$\boldsymbol{E}(\boldsymbol{r}) = -\boldsymbol{\nabla} V(\boldsymbol{r}). \tag{3}$$

Introducing Eq. (2) into the above equation gives

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) &= -\frac{1}{4\pi\varepsilon_0}\boldsymbol{\nabla}\left(\frac{\boldsymbol{p}\cdot\boldsymbol{R}}{R^3}\right)\\ &= -\frac{1}{4\pi\varepsilon_0}p_j\boldsymbol{\nabla}\left(\frac{R_j}{R^3}\right).\end{aligned} \tag{4}$$

To calculate this expression we look closer at the following two expressions:

$$\partial_i R = \partial_i\sqrt{R_k R_k} = \frac{R_i}{R}, \tag{5}$$

and

$$\begin{aligned}p_j\partial_i\left(\frac{R_j}{R^3}\right) &= p_j\frac{(\partial_i R_j)R^3 - R_j(\partial_i R^3)}{R^6}\\ &= p_j\frac{\delta_{ij}R^3 - R_j(3R^2(\partial_i R)}{R^6}\\ &= p_j\left[\frac{\delta_{ij}}{R^3} - 3\frac{R_j R_i}{R^5}\right]\\ &= \frac{p_i}{R^3} - 3\frac{(\boldsymbol{p}\cdot\boldsymbol{R})R_i}{R^5}.\end{aligned} \tag{6}$$

Substituting this latter expression back into Eq. (4) gives after introducing unit position vectors

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{3(\boldsymbol{p}\cdot\hat{\boldsymbol{R}})\hat{\boldsymbol{R}}-\boldsymbol{p}}{R^3}, \qquad \boldsymbol{R}=\boldsymbol{r}-\boldsymbol{r}'. \tag{7}$$

This is the final expression for the electric field in coordinate free form.

Problem 2.
See Griffiths!

Problem 3.
See Griffiths!