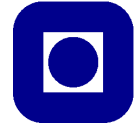


TFY4240

Solution problemset 4 Autumn 2014

NTNU

Institutt for
fysikk**Problem 1.**

- a) In the lectures the electric dipole was placed at the center of the coordinate system and one obtained

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}, \quad (1)$$

for the scalar electric potential. In a coordinate system not located at the center of the dipole, \mathbf{r} therefore has to be replaced by the distance from the dipole to the observation point. If we denote this distance by \mathbf{R} one thus obtains

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{R}}}{R^2}, \quad \mathbf{R} = \mathbf{r} - \mathbf{r}', \quad (2)$$

which is valid in any coordinate system. Formally the above expression is obtained by a change of spacial variable from the “centered” to the “non-centered” coordinate system.

- b) The electric field is obtained in the standard way from

$$\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r}). \quad (3)$$

Introducing Eq. (2) into the above equation gives

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= -\frac{1}{4\pi\epsilon_0} \nabla \left(\frac{\mathbf{p} \cdot \mathbf{R}}{R^3} \right) \\ &= -\frac{1}{4\pi\epsilon_0} p_j \nabla \left(\frac{R_j}{R^3} \right). \end{aligned} \quad (4)$$

To calculate this expression we look closer at the following two expressions:

$$\partial_i R = \partial_i \sqrt{R_k R_k} = \frac{R_i}{R}, \quad (5)$$

and

$$\begin{aligned} p_j \partial_i \left(\frac{R_j}{R^3} \right) &= p_j \frac{(\partial_i R_j) R^3 - R_j (\partial_i R^3)}{R^6} \\ &= p_j \frac{\delta_{ij} R^3 - R_j (3R^2 (\partial_i R))}{R^6} \\ &= p_j \left[\frac{\delta_{ij}}{R^3} - 3 \frac{R_j R_i}{R^5} \right] \\ &= \frac{p_i}{R^3} - 3 \frac{(\mathbf{p} \cdot \mathbf{R}) R_i}{R^5}. \end{aligned} \quad (6)$$

Substituting this latter expression back into Eq. (4) gives after introducing unit position vectors

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{3(\mathbf{p} \cdot \hat{\mathbf{R}})\hat{\mathbf{R}} - \mathbf{p}}{R^3}, \quad \mathbf{R} = \mathbf{r} - \mathbf{r}'. \quad (7)$$

This is the final expression for the electric field in coordinate free form.

Problem 2.

See Griffiths!

Problem 3.

See Griffiths!