TFY4240

Solution problemset 5 Autumn 2014

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Problem 1.

a) The magnetic induction is related to the vector potential by

$$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla} \times \boldsymbol{A}(\boldsymbol{r}). \tag{1}$$

Hence, it is in principle straight forward to obtain $\boldsymbol{B}(\boldsymbol{r})$.

In component form we therefore get,

$$\begin{aligned} B_i(\boldsymbol{r}) &= \varepsilon_{ijk}\partial_j A_k(\boldsymbol{r}) \\ &= \frac{\mu_0}{4\pi}\varepsilon_{ijk}\partial_j\left(\varepsilon_{klm}\frac{m_l r_m}{r^3}\right) \\ &= \frac{\mu_0}{4\pi}\varepsilon_{kij}\varepsilon_{klm}\frac{m_l\delta_{jm}r^3 - m_l r_m 3 r r_j}{r^6}, \end{aligned} \tag{2}$$

where we have used that $\partial_j r^3 = 3rr_j$ and $\varepsilon_{ijk} = \varepsilon_{kij}$. Now expanding the term $\varepsilon_{kij}\varepsilon_{klm}$ gives

$$\begin{aligned} B_i(\boldsymbol{r}) &= \frac{\mu_0}{4\pi}\left(\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}\right)\left[\frac{m_l\delta_{jm}}{r^3} - 3\frac{m_l r_m r_j}{r^5}\right] \\ &= \frac{\mu_0}{4\pi}\left[\frac{3m_i}{r^3} - \frac{m_i}{r^3} - 3\frac{m_i r_j r_j}{r^5} + 3\frac{m_j r_i r_j}{r^5}\right] \\ &= \frac{\mu_0}{4\pi}\left[3\frac{(\boldsymbol{m}\cdot\boldsymbol{r})\,r_i}{r^5} - \frac{m_i}{r^3}\right]. \end{aligned} \tag{3}$$

Here in the second transition, we have used that $\delta_{jm}\delta_{jm} = 3$ and $m_j\delta_{ji} = m_i$.

Hence, we finally obtain for the magnetic induction

$$\begin{aligned} \boldsymbol{B}(\boldsymbol{r}) &= \frac{\mu_0}{4\pi}\left[3\frac{(\boldsymbol{m}\cdot\boldsymbol{r})\,\boldsymbol{r}}{r^5} - \frac{\boldsymbol{m}}{r^3}\right] \\ &= \frac{\mu_0}{4\pi}\frac{3\,(\boldsymbol{m}\cdot\hat{\boldsymbol{r}})\,\hat{\boldsymbol{r}} - \boldsymbol{m}}{r^3}. \end{aligned} \tag{4}$$

b) Equation (4) is similar to the expression for the electric field around an electric dipole under the substitutions $\boldsymbol{E} \to \boldsymbol{B}$ and $\boldsymbol{p} \to \boldsymbol{m}$.

Problem 2.

a) The field from a current element $Id\boldsymbol{l}$ is given by Biot-Savart's law

$$d\boldsymbol{H} = \frac{1}{4\pi}\frac{Id\boldsymbol{l}\times\hat{\boldsymbol{r}}}{r^2} \tag{5}$$

At position O, there is no contribution to the magnetic field from the straight parts of the wire, since $d\boldsymbol{l} \times \hat{\boldsymbol{r}} = 0$.

Hence the total magnetic field at O becomes (where the integration is over the semi-circle)

$$\boldsymbol{H} = \int d\boldsymbol{H} = \hat{\boldsymbol{z}} \frac{I}{4\pi R} \int_0^\pi d\theta = \underline{\frac{I}{4R}\hat{\boldsymbol{z}}} \qquad (6)$$

where we have used that $dl = Rd\theta$

b)

$$|\boldsymbol{H}| = \frac{I}{4R} = 0.25 \cdot 10^2 \text{A/m} = \underline{25 \text{A/m}} \qquad (7)$$

The direction of the field is <u>out</u> of the paper-plane.

Problem 3.
See Griffiths!