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Institutt for fysikk

# TFY4240

# Solution problemset 6 Autumn 2014

**Problem 1.**
See Griffiths!

**Problem 2.**

**a)** The current density is only non-zero along the infinite wire, so we get

$$\boldsymbol{J}(\boldsymbol{r}) = I\delta(x)\delta(y)\hat{\boldsymbol{z}}. \tag{1}$$

**b)** From Biot-Savarts law it follows that ($\boldsymbol{R} = \boldsymbol{r}$ - $\boldsymbol{r'}$)

$$\begin{aligned}\boldsymbol{B}(\boldsymbol{r}) =& \frac{\mu_0}{4\pi}\int \mathrm{d}^3r' \frac{\boldsymbol{J}(\boldsymbol{r'}) \times \hat{\boldsymbol{R}}}{R^2} && (2)\\ =& \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} \mathrm{d}z' \frac{\hat{\boldsymbol{z}}' \times \hat{\boldsymbol{R}}}{|\boldsymbol{r} - z'\hat{\boldsymbol{z}}|^2} && (3)\\ =& \frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\phi}} && (4)\end{aligned}$$

Here $s$ is the distance from the long wire to the observation point. The last integral was calculated using the method of Example 5.5 in Griffiths.

**c)** We first find the flux through the loop:

$$\begin{aligned}\boldsymbol{\Phi} = \int \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{a} =& \frac{\mu_0 I}{2\pi}\int_{s_0}^{s_0+a} \frac{1}{s} a\mathrm{d}s && (5)\\ =& \frac{\mu_0 I a}{2\pi}\ln\left(\frac{s_0+a}{s_0})\right) && (6)\end{aligned}$$

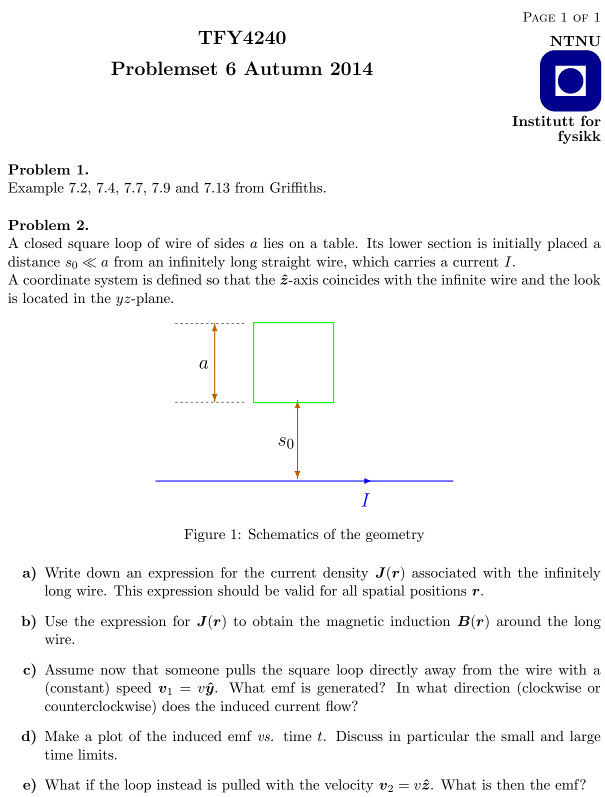
By introducing a time-dependent $s$, we get

$$\boldsymbol{\Phi}(t) = \frac{\mu_0 I a}{2\pi}\ln(\frac{s(t)+a}{s(t)}), \tag{7}$$

where in our case $s(t) = s_0 + vt$, since $\boldsymbol{v} = v\hat{\boldsymbol{y}}$.

The generated emf $\varepsilon$ can now be calculated from the formula

$$\begin{aligned}\varepsilon = -\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} =& -\frac{\mu_0 I a}{2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\ln(\frac{s(t)+a}{s(t)}) && (8)\\ =& -\frac{\mu_0 I a}{2\pi}\left[\frac{1}{s(t)+a}\frac{\mathrm{d}s(t)}{\mathrm{d}t} - \frac{1}{s(t)}\frac{\mathrm{d}s(t)}{\mathrm{d}t}\right] && (9)\\ =& \underline{\frac{\mu_0 I v a^2}{2\pi s(t)(s(t)+a)}} && (10)\end{aligned}$$



Page 1 of 1

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**TFY4240**

**Problemset 6 Autumn 2014**

**Problem 1.**
Example 7.2, 7.4, 7.7, 7.9 and 7.13 from Griffiths.

**Problem 2.**
A closed square loop of wire of sides $a$ lies on a table. Its lower section is initially placed a distance $s_0 \ll a$ from an infinitely long straight wire, which carries a current $I$.
A coordinate system is defined so that the $\hat{\boldsymbol{z}}$-axis coincides with the infinite wire and the look is located in the $yz$-plane.



Figure 1: Schematics of the geometry

**a)** Write down an expression for the current density $\boldsymbol{J}(\boldsymbol{r})$ associated with the infinitely long wire. This expression should be valid for all spatial positions $\boldsymbol{r}$.

**b)** Use the expression for $\boldsymbol{J}(\boldsymbol{r})$ to obtain the magnetic induction $\boldsymbol{B}(\boldsymbol{r})$ around the long wire.

**c)** Assume now that someone pulls the square loop directly away from the wire with a (constant) speed $\boldsymbol{v}_1 = v\hat{\boldsymbol{y}}$. What emf is generated? In what direction (clockwise or counterclockwise) does the induced current flow?

**d)** Make a plot of the induced emf *vs.* time $t$. Discuss in particular the small and large time limits.

**e)** What if the loop instead is pulled with the velocity $\boldsymbol{v}_2 = v\hat{\boldsymbol{z}}$. What is then the emf?

Figure 1: The emf *vs.* time

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where we in the last step have used that $\frac{\mathrm{d}s(t)}{\mathrm{d}t} = v$.

The field points out of the page, so the force on a charge in the segment of the loop closest to the infinite wire is to the right. The force on a charge on the far side of the loop is also to the right, but here the field and therefore the force is weaker. The current in the loop is therefore flowing counterclockwise. (This can also be verified from Lenz' law: Since the flux pointing out of the page is getting weaker as we move the loop away, the induced current will give a flux also pointing out of the page, which means that the current must flow counterclockwise.)

**d)** See Fig. 1.

**e)** Now the flux is constant, so $\varepsilon = \frac{\mathrm{d}\Phi}{\mathrm{d}t} = 0$.