NTNU

Institutt for fysikk

TFY4240

Solution problemset 7 Autumn 2014

Problem 1.

a) The flux is given by

$$\phi = \int_0^x B_n dA = \int_0^x B\frac{1}{2}\sqrt{2}l ds = \frac{1}{2}\sqrt{2}l \int_0^x B ds, \tag{1}$$

$$\phi(x) = \frac{1}{2}\sqrt{2}lBx. \tag{2}$$

The induced emf is given by

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d\phi}{dx}\frac{dx}{dt} = -\frac{1}{2}\sqrt{2}lB\frac{dx}{dt} = -\frac{1}{2}\sqrt{2}lBv. \tag{3}$$

The current is flowing clockwise, and the current is given by

$$I = \frac{|\varepsilon|}{R} = \frac{\frac{1}{2}\sqrt{2}lBv}{R}. \tag{4}$$

b) The force is given by

$$\boldsymbol{F} = I\boldsymbol{l} \times \boldsymbol{B}, \tag{5}$$

$$|\boldsymbol{F}| = IlB = \frac{\frac{1}{2}\sqrt{2}l^2B^2v}{R} \tag{6}$$

and has the components

$$F_y = -F_x = \frac{\frac{1}{2}l^2B^2v}{R}. \tag{7}$$

The mechanical effect is given by

$$P = |F_x|v = \frac{\frac{1}{2}l^2B^2v^2}{R}, \tag{8}$$

and the electrical effect

$$P_E = RI^2 = R\left(\frac{\frac{1}{2}lBv}{R}\right)^2 = \frac{\frac{1}{2}l^2B^2v^2}{R}. \tag{9}$$

Hence

$$P = P_E \tag{10}$$

c) The vertical component of the force is given by

$$F_y = \frac{\frac{1}{2}l^2 B^2 v}{R}. \tag{11}$$

The rod levitates when this force is larger than the gravity force. This happens when

$$\frac{\frac{1}{2}l^2 B^2 v}{R} \geq mg, \tag{12}$$

or

$$B^2 \geq \frac{2mgR}{l^2 v}. \tag{13}$$

d) Assume $\boldsymbol{B} \perp \boldsymbol{J}$ and that the orientation is such that the force acts vertical

$$|F| = BIl \geq mg \tag{14}$$

$$I \geq \frac{mg}{Bl} = 4905A. \tag{15}$$

The current must flow in the opposite direction of the current density in the plane,

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}, \tag{16}$$

where $qv = I$.

Problem 2.

a) The components are

$$\begin{aligned} \boldsymbol{v} &= v_i\, \hat{\boldsymbol{x}}_i, \\ \bar{\boldsymbol{t}} &= t_{ij}\, \hat{\boldsymbol{x}}_i \otimes \hat{\boldsymbol{x}}_j, \\ \overline{\boldsymbol{T}} &= T_{ij}\, \hat{\boldsymbol{x}}_i \otimes \hat{\boldsymbol{x}}_j, \end{aligned} \tag{17}$$

b) Using the relations from Eq. (17), one obtains

$$\begin{aligned} \overline{\boldsymbol{T}} &= [v_i\, \hat{\boldsymbol{x}}_i] \times [t_{mn}\, \hat{\boldsymbol{x}}_m \otimes \hat{\boldsymbol{x}}_n] \\ &= v_i t_{mn} \underbrace{\hat{\boldsymbol{x}}_i \times (\hat{\boldsymbol{x}}_m \otimes \hat{\boldsymbol{x}}_n)}_{\underbrace{(\hat{\boldsymbol{x}}_i \times \hat{\boldsymbol{x}}_m)}_{\varepsilon_{kim}\, \hat{\boldsymbol{x}}_k} \otimes \hat{\boldsymbol{x}}_n} \\ &= \varepsilon_{kim} v_i t_{mn}\, \hat{\boldsymbol{x}}_k \otimes \hat{\boldsymbol{x}}_n. \end{aligned} \tag{18}$$

Combining this with the results from subproblem a) one finds

$$T_{kn} = \varepsilon_{kim} v_i t_{mn}, \tag{19}$$

or after renaming dummy indices

$$T_{ij} = \varepsilon_{ikl} v_k t_{lj}. \tag{20}$$

c) To calculate $\boldsymbol{\nabla}\cdot\overline{F}(\boldsymbol{r})$ one has to keep in mind that $\boldsymbol{\nabla} = \hat{\boldsymbol{x}}_i\partial_i$ is a vector values operator that operate on what appears on its right. With this in mind one gets

$$\begin{aligned}
\boldsymbol{\nabla}\cdot\overline{F}(\boldsymbol{r}) &= [\hat{\boldsymbol{x}}_i\partial_i]\cdot[F_{mn}(\boldsymbol{r})\,\hat{\boldsymbol{x}}_m\otimes\hat{\boldsymbol{x}}_n] \\
&= [\partial_i F_{mn}(\boldsymbol{r})]\;\hat{\boldsymbol{x}}_i\cdot(\hat{\boldsymbol{x}}_m\otimes\hat{\boldsymbol{x}}_n) \\
&= [\partial_i F_{mn}(\boldsymbol{r})]\;\underbrace{(\hat{\boldsymbol{x}}_i\cdot\hat{\boldsymbol{x}}_m)}_{\delta_{im}}\,\hat{\boldsymbol{x}}_n \\
&= [\partial_i F_{in}(\boldsymbol{r})]\;\hat{\boldsymbol{x}}_n.
\end{aligned} \tag{21}$$

Hence, component i of the vector $\boldsymbol{\nabla}\cdot\overline{F}(\boldsymbol{r})$ becomes

$$\left[\boldsymbol{\nabla}\cdot\overline{F}(\boldsymbol{r})\right]_i = \partial_j F_{ji}(\boldsymbol{r}). \tag{22}$$