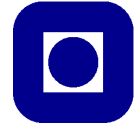


TFY4240

Solution problemset 7 Autumn 2014

NTNU

Institutt for
fysikk**Problem 1.**

a) The flux is given by

$$\phi = \int_0^x B_n dA = \int_0^x B \frac{1}{2} \sqrt{2} l ds = \frac{1}{2} \sqrt{2} l \int_0^x B ds, \quad (1)$$

$$\phi(x) = \frac{1}{2} \sqrt{2} l B x. \quad (2)$$

The induced emf is given by

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d\phi}{dx} \frac{dx}{dt} = -\frac{1}{2} \sqrt{2} l B \frac{dx}{dt} = -\frac{1}{2} \sqrt{2} l B v. \quad (3)$$

The current is flowing clockwise, and the current is given by

$$I = \frac{|\varepsilon|}{R} = \frac{\frac{1}{2} \sqrt{2} l B v}{R}. \quad (4)$$

b) The force is given by

$$\mathbf{F} = I \mathbf{l} \times \mathbf{B}, \quad (5)$$

$$|\mathbf{F}| = I l B = \frac{\frac{1}{2} \sqrt{2} l^2 B^2 v}{R} \quad (6)$$

and has the components

$$F_y = -F_x = \frac{\frac{1}{2} l^2 B^2 v}{R}. \quad (7)$$

The mechanical effect is given by

$$P = |F_x| v = \frac{\frac{1}{2} l^2 B^2 v^2}{R}, \quad (8)$$

and the electrical effect

$$P_E = R I^2 = R \left(\frac{\frac{1}{2} l B v}{R} \right)^2 = \frac{\frac{1}{2} l^2 B^2 v^2}{R}. \quad (9)$$

Hence

$$P = P_E \quad (10)$$

c) The vertical component of the force is given by

$$F_y = \frac{\frac{1}{2}l^2 B^2 v}{R}. \quad (11)$$

The rod levitates when this force is larger than the gravity force. This happens when

$$\frac{\frac{1}{2}l^2 B^2 v}{R} \geq mg, \quad (12)$$

or

$$B^2 \geq \frac{2mgR}{l^2 v}. \quad (13)$$

d) Assume $\mathbf{B} \perp \mathbf{J}$ and that the orientation is such that the force acts vertical

$$|F| = BIl \geq mg \quad (14)$$

$$I \geq \frac{mg}{Bl} = 4905A. \quad (15)$$

The current must flow in the opposite direction of the current density in the plane,

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}, \quad (16)$$

where $qv = I$.

Problem 2.

a) The components are

$$\begin{aligned} \mathbf{v} &= v_i \hat{\mathbf{x}}_i, \\ \bar{\mathbf{t}} &= t_{ij} \hat{\mathbf{x}}_i \otimes \hat{\mathbf{x}}_j, \\ \bar{\mathbf{T}} &= T_{ij} \hat{\mathbf{x}}_i \otimes \hat{\mathbf{x}}_j, \end{aligned} \quad (17)$$

b) Using the relations from Eq. (17), one obtains

$$\begin{aligned} \bar{\mathbf{T}} &= [v_i \hat{\mathbf{x}}_i] \times [t_{mn} \hat{\mathbf{x}}_m \otimes \hat{\mathbf{x}}_n] \\ &= v_i t_{mn} \underbrace{\hat{\mathbf{x}}_i \times (\hat{\mathbf{x}}_m \otimes \hat{\mathbf{x}}_n)}_{\underbrace{(\hat{\mathbf{x}}_i \times \hat{\mathbf{x}}_m)}_{\varepsilon_{kim}} \otimes \hat{\mathbf{x}}_n} \\ &= \varepsilon_{kim} v_i t_{mn} \hat{\mathbf{x}}_k \otimes \hat{\mathbf{x}}_n. \end{aligned} \quad (18)$$

Combining this with the results from subproblem a) one finds

$$T_{kn} = \varepsilon_{kim} v_i t_{mn}, \quad (19)$$

or after renaming dummy indices

$$T_{ij} = \varepsilon_{ikl} v_k t_{lj}. \quad (20)$$

- c) To calculate $\nabla \cdot \bar{F}(\mathbf{r})$ one has to keep in mind that $\nabla = \hat{\mathbf{x}}_i \partial_i$ is a vector values operator that operate on what appears on its right. With this in mind one gets

$$\begin{aligned}
 \nabla \cdot \bar{F}(\mathbf{r}) &= [\hat{\mathbf{x}}_i \partial_i] \cdot [F_{mn}(\mathbf{r}) \hat{\mathbf{x}}_m \otimes \hat{\mathbf{x}}_n] \\
 &= [\partial_i F_{mn}(\mathbf{r})] \hat{\mathbf{x}}_i \cdot (\hat{\mathbf{x}}_m \otimes \hat{\mathbf{x}}_n) \\
 &= [\partial_i F_{mn}(\mathbf{r})] \underbrace{(\hat{\mathbf{x}}_i \cdot \hat{\mathbf{x}}_m)}_{\delta_{im}} \hat{\mathbf{x}}_n \\
 &= [\partial_i F_{in}(\mathbf{r})] \hat{\mathbf{x}}_n.
 \end{aligned} \tag{21}$$

Hence, component i of the vector $\nabla \cdot \bar{F}(\mathbf{r})$ becomes

$$[\nabla \cdot \bar{F}(\mathbf{r})]_i = \partial_j F_{ji}(\mathbf{r}). \tag{22}$$