TFY4240

Solution problemset 8 Autumn 2014

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Problem 1.

a) Direct calculation results in

$$\begin{aligned}\langle\cos(\omega t)\rangle_t &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} dt\,\cos(\omega t)\\ &= \lim_{T\to\infty}\frac{1}{T}\left[\frac{\sin(\omega t)}{\omega}\right]_{-T/2}^{T/2}\\ &= \lim_{T\to\infty}\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}\\ &= 0.\end{aligned} \tag{1}$$

In the same way one can show that $\langle\sin(\omega t)\rangle_t = 0$.

b) To show that $\left\langle\cos^2(\omega t)\right\rangle_t = \left\langle\sin^2(\omega t)\right\rangle_t = 1/2$ we start by using the identities

$$\cos^2(\omega t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega t) \tag{2a}$$

$$\sin^2(\omega t) = \frac{1}{2} - \frac{1}{2}\cos(2\omega t), \tag{2b}$$

and that the average of the last terms of the above equations are zero as shown in the previous sub-problem.

The final result follows trivially by noting that

$$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} dt\,\frac{1}{2} = \frac{1}{2}. \tag{3}$$

Problem 2.

a) See the lecture notes....

b) At the first interface, between media 0 and 1, the incident wave E_i is partly reflected and partly transmitted. The transmitted part is then partly reflected and partly transmitted at the interface between media 1 and 2, and so on. The total reflected and total transmitted waves thus becomes sums of partial waves. By adding together the amplitudes, we get for the reflection coefficient

$$
\begin{aligned}
r &= r_{01} + t_{01}r_{12}t_{10}e^{2i\delta_1} + t_{01}r_{12}r_{10}r_{12}t_{10}e^{4i\delta_1} \\
&+ ... + t_{01}r_{12}t_{10}(r_{12}r_{10})^n e^{2(n+1)i\delta_1} \\
&= r_{01} + t_{01}r_{12}t_{10}e^{2i\delta_1}\sum_{k=0}^{\infty}(r_{12}r_{10}e^{2i\delta_1})^k \\
&= r_{01} + \frac{t_{01}r_{12}t_{10}e^{2i\delta_1}}{1 - r_{12}r_{10}e^{2i\delta_1}} = r_{01} + \frac{t_{01}r_{12}t_{10}e^{2i\delta_1}}{1 + r_{12}r_{01}e^{2i\delta_1}} \\
&= \frac{r_{01} + r_{12}r_{01}^2e^{2i\delta_1} + t_{01}r_{12}t_{10}e^{2i\delta_1}}{1 + r_{12}r_{01}e^{2i\delta_1}} = \frac{r_{01} + r_{12}e^{2i\delta_1}(r_{01}^2 + t_{01}t_{10})}{1 + r_{12}r_{01}e^{2i\delta_1}} \\
&= \frac{r_{01} + r_{12}e^{2i\delta_1}}{1 + r_{12}r_{01}e^{2i\delta_1}}
\end{aligned}
$$

where in the last line it is used that $r_{01}^2 + t_{01}t_{10} = 1$. Similarily, we get for the transmission coefficient

$$
\begin{aligned}
t &= t_{01}t_{12}e^{i\delta_1} + t_{01}r_{12}r_{10}t_{12}e^{3i\delta_1} + ... \\
&= t_{01}t_{12}e^{i\delta_1}\sum_{k=0}^{\infty}(r_{12}r_{10}e^{2i\delta_1})^k \\
&= \frac{t_{01}t_{12}e^{i\delta_1}}{1 + r_{12}r_{01}e^{2i\delta_1}}
\end{aligned}
$$

c) The phase difference between the the component reflected at the interface between media 0 and 1 and the component reflected at the interface between media 1 and 2 , is $2\delta_1$. The difference in optical pathlength s can be found from Figure 1:

$$
\begin{aligned}
s &= 2n_1\frac{d}{\cos\phi_1} - 2d\tan\phi_1\; n_0\sin\phi_0 \\
&= 2d\left(\frac{n_1}{\cos\phi_1} - \frac{\sin\phi_1}{\cos\phi_1}n_1\sin\phi_1\right) \\
&= 2dn_1\cos\phi_1
\end{aligned}
$$

This gives

$$
\delta_1 = \frac{2\pi}{\lambda}n_1 d\cos\phi_1. \tag{4}
$$

d) Assuming normal incidence, we have the reflection coefficients

$$
\begin{aligned}
r_{01} &= \frac{n_0 - n_1}{n_0 + n_1} \\
r_{12} &= \frac{n_1 - n_2}{n_1 + n_2}
\end{aligned} \tag{5}
$$

Insert this into

$$\frac{r_{01}+r_{12}}{1+r_{01}r_{12}} = \frac{\left(\frac{n_0-n_1}{n_0+n_1}\right)+\left(\frac{n_1-n_2}{n_1+n_2}\right)}{1+\left(\frac{n_0-n_1}{n_0+n_1}\right)\left(\frac{n_1-n_2}{n_1+n_2}\right)}$$
$$= \frac{2n_1(n_0-n_2)}{2n_1(n_0+n_2)} = \frac{n_0-n_2}{n_0+n_2} = r_{02}$$

This must also be so from the equation for a thin film on a surface:

$$r = \frac{r_{01}+r_{12}e^{2i\delta_1}}{1+r_{12}r_{01}e^{2i\delta_1}}$$

If the thin film thickness is zero, we have $\delta_1 = 0$ and

$$r = \frac{r_{01}+r_{12}}{1+r_{01}r_{12}}.$$

This corresponds to the reflection at the 0-2 interface.

e) If $2\delta_1 = 2\pi$ the wall becomes "invisible" for the radiation (that follows by comparing Eqs.(1) and (3) from the problem set)

$$\delta_1 = \frac{2\pi}{\lambda} n_1 d \cos\phi_1 \qquad \phi_1 = 0$$
$$2\pi = 2\frac{2\pi}{\lambda} n_1 d$$
$$d = \frac{\lambda}{2n_1} = \frac{c}{2\nu n_1} = \frac{3\cdot 10^8}{2\cdot 10^{10}\cdot 2.5} = 0.6\cdot 10^{-2}\text{m} = 6\text{mm}$$