TFY4275 Classical Transport Theory

Problemset 1 Spring 2013

NTNU

Institutt for fysikk

Problem 1. Finance: Distribution of returns

This problem is devoted to the determination of the tails of an empirical distribution. We will consider the major American stock index known as the Dow Jones Industrial Average (DJIA). It consists of a small number of major companies (today a little more then 30) from various sectors of the industry, like, *e.g.* Microsoft and Exxon. It is often used as a gauge of how well the American industry performs.

Download the data-set : http://web.phys.ntnu.no/~ingves/Teaching/TFY4275/Data/djia-closing.dat.gz. It contains the historic daily (closing) values for the DJIA over the time period from 1896 till 2001. It is this data set that we will be analyzing in this problem.

If the value of the index is denoted $P(t)$, the so-called *logarithmic return* is defined as:

$$r_{\Delta t}(t) \quad = \quad \ln\left(\frac{P(t+\Delta t)}{P(t)}\right), \tag{1}$$

where Δt is a time period that we here will take to be one day (*i.e.* we consider the daily returns).

a) Plot the DJIA index, and notice in particular the famous index crashes of 1929 and 1987 *etc.* Due to the increase of the index with time, you might find it useful to use a logarithmic scale on the 2nd axes.

b) Calculate, and plot, the daily (logarithmic) returns as defined by Eq. (1). Do you observe any periodicity in this time-series? If so, how do you explain this behavior.

c) Calculate the standard deviation, $\sigma_{\Delta t}$, of stochastic process $r_{\Delta t}(t)$. In finance this quantity is known as the *volatility*, and is used as a (simple) measure of risk.

d) Try to empirically determine the pdf of returns, $p(r_{\Delta t})$ (for $\Delta t = 1$). Remember that the pdf should be normalized to one! [Here you will have to use a binning]. Compare your findings to the corresponding Gaussian distribution of a similar standard deviation, by plotting both of them in the same graph. What is the main difference between the two, and how do you interpret this difference?

e) Try now to determine the cumulative distribution functions $P(r_{\Delta t})$ and $1 - P(r_{\Delta t})$. Compare the tails of the two.

f) Optional : It is often claimed that $1 - P(r_{\Delta t}) \sim r_{\Delta t}^{-(1+\alpha)}$ $(\alpha > 0)$ over some range of return values, *i.e.* it shows power-law scaling. Estimate the value of the power-law index α.