TFY4275 Classical Transport Theory
Problemset 2 Spring 2013

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Problem 1. Discrete Random Walk

In this problem, we will study a *Discrete Random Walk* with a continuous jump size distribution. As in the lectures, we will denote the position of the walker at (discrete) time t by $x(t)$. It is obtained as a sum of independent jumps

$$x(t) \quad = \quad \sum_{i=1}^{t} \xi_i, \qquad t = 1, 2, 3, \dots, \tag{1}$$

where the size of the jumps ξ_i are assumed to be *statistically independent* and distributed uniformly in the interval between -1 and 1, *i.e.* $\xi = U(-1, 1)$. In writing Eq. (1), we have assumed that $x(t = 0) = 0$.

a) What is the characteristic function of the uniform jump pdf $p_\xi(\xi)$?

b) Use the convolution theorem of probabilities to calculate the probability distribution function (pdf) for $x(t)$ for $t = 2$? We will denote this probability by $p(x, t = 2)$.

c) Calculate $p(x, t = 2)$ using the characteristic function from question a.

d) Obtain the characteristic function, $G_x(k; t)$, valid for *any* time t.

e) Use this result to calculate the probability density function, $p(x; t)$, of the walker for an arbitrary time t. [You may have to do this numerically.]

f) Plot $p(x; t)$ *vs.* x for small and large times $t = 2, 5, 10, 50$ and 100.

g) Calculate the 1st and 2nd moment of the pdf $p(x; t)$.

h) Use these results to calculate the standard deviation of the position of the random walker.

i) Plot the rescaled probability $p(x; t)\sigma(t)$ *vs.* x for the same values of t as used above. Based on this result, what do you think will happen when $t \to \infty$? How do you explain this result.

Problem 2. Numerical Implementation of the Discrete Random Walk

In this exercises you are asked to perform a simulation of the discrete random walk model mentioned above. For this purpose, you can use whatever computer program that you are familiar with; *e.g.* Matlab (or the open source clone Octave) will quite nicely do the job.

a) Write a routine (and main program) for implementing the random walk model from the previous problem. You should specify the time variable t (the input), and the routine will output *one* realization of the walker.

b) Plot this realization as position $x(t)$ *vs.* time t in a xy-plot.

c) In the same graph, plot a few different realization of the random walker. [*If you end up with the same path, you will have to change the so-called seed of your random number generator*].

d) Optional : Write a routine that can calculate the standard deviation, $\sigma(t)$, of the position of the walker up to a maximum time t. This quantity has to be averaged over an ensemble of realizations, N, with a suitable sampling of time (up to, say, $t = 100$). The number of samples has to be large enough to make sure that the results have converged.

e) Compare these numerical results for $\sigma(t)$ to what you obtained theoretically in the previous problem.