TFY4275 Classical Transport Theory
Problemset 3 Spring 2013

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Problem 1.
In the lectures we used the characteristic function, $G(k)$, of the Gaussian (Normal) distribution,

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\},$$

where μ and σ denote the mean and the standard deviation of the distribution, respectively. In this problem you are asked to show that this characteristic function reads

$$G(k) = \exp\left(ik\mu - \frac{\sigma^2 k^2}{2} \right).$$

Problem 2.
In the first exercise of this class, we used the historic financial time series of the DJIA as an example. In this problem we will again look at the same data set.
From the historic prices $P(t)$, we defined the daily (logarithmic) returns as

$$r_{\Delta t}(t) = \ln\left(\frac{P(t+\Delta t)}{P(t)} \right),$$

where Δt is a time period.

a) Take (or repeat) the calculation for the pdf of returns, $p(r_{\Delta t})$. [Exercise 1].

b) Show that

$$r_{n\Delta t}(t) = \sum_{j=0}^{n-1} r_{\Delta t}(t + j\Delta t).$$

That is, the (logarithmic) returns over a time window $n\Delta t$ is the same as the sum of the n consecutive (logarithmic) returns over time window Δt starting at time t.

c) The above result is general, and does not assume anything about *e.g.* statistical independence of the returns $r_{\Delta t}$. We will, however, make this assumption herein. Use $p(r_{\Delta t})$ to calculate $p(r_{n\Delta t})$ for some values of n, say (*i*) $n = 2$; (*ii*) 10; (*iii*) 100 and (*iv*) 500. What can you say about the distribution $p(r_{n\Delta t})$ for lager n?

d) Use the definition of the returns given above to calculates directly the distributions $p(r_{n\Delta t})$.

e) The widths of the distributions, $p(r_{n\Delta t})$ (calculated with the direct method), are often used as a measure of financial risk (it is called the volatility). What can you say about the "risk" for increasing n (time)?

f) Compare the results for the distributions $p(r_{n\Delta t})$ obtained by the two methods. What do you think about our initial assumption that $r_{\Delta t}(t)$ for different times are statistical independent?