TFY4275 Classical Transport Theory
Problemset 5 Spring 2013

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Problem 1.
We will here study the cumulants of the Gaussian distribution. Calculate for this distribution the first two cumulants, κ_1 and κ_2, and express them in terms of the mean (μ) and standard deviation (σ).
Moreover, show that all cumulants of order three or higher are zero for the Gaussian distribution, *i.e.* $\kappa_n = 0$ when $n \geq 3$.
The above property makes the cumulants particularly useful for the Gaussian distribution (where *e.g.* $\langle \exp(\xi) \rangle$ takes on a simple form).
[Hint: Use the cumulant generating function $\ln G(k)$]

Problem 2.
Also in this exercise, we will study the historical data of the American DJIA index. As before, we define the log-returns by

$$r_{\Delta t}(t) \quad = \quad \ln\left(\frac{P(t+\Delta t)}{P(t)}\right), \tag{1}$$

where $P(t)$ denotes the value of the index at time t. We will now study various return correlation functions of this time series for $\Delta t = 1$ day.

a) Calculate by, *e.g.* Matlab (or Octave), the return-return correlations function

$$W_1(t) \quad = \quad \frac{\langle r_{\Delta t}(t'+t) r_{\Delta t}(t') \rangle_{t'}}{\sigma_r^2}. \tag{2}$$

for time a relatively large interval, *e.g.* $t \in [0, 1000]$. In Eq. (2), σ_r denotes the standard deviation of the return time series. It is called daily volatility in finance, and it is (here) introduced to normalize the correlation function properly so that $W_1(0) = 1$.

How long would you estimate the return correlation time to be?

[Note that there will typically be some not-insignificant noise for small levels of correlations. This is normal when working with empirical data. So zero correlation, does not necessarily mean that the empirical correlation function is zero. For this one needs an infinitely long time series (or an ensemble of time series).]

b) Calculate a similar correlation function, but now for the *square* of the return $(r_{\Delta t}(t))^2 = r_{\Delta t}^2(t)$, *i.e.* calculate

$$W_2(t) \quad \propto \quad \left\langle r_{\Delta t}^2(t'+t) r_{\Delta t}^2(t') \right\rangle_{t'}. \tag{3}$$

What should the proper normalization constant of the correlation function be in this case?

What would you say is the correlation time in this case?

c) Compare the two correlation time estimates that you obtained above. Are they similar or different? Can you give some financial argument in favor of this observation?