# TFY4275 Classical Transport Theory

# Problemset 6 Spring 2013

NTNU

Institutt for fysikk

**Problem 1.**

This is the story of Prof. Baltazar (for those of you that recall him from your childhood). He owns a cosy cabin up in the mountains located next to a *calm*, *straight* and *narrow* river. He often goes there to enjoy the fresh air and the clear and calm water. However, one day, one of his neighbors informs him about a small fuel spill ($M = 1kg$) he had at his place located about $L = 200m$ up the river. Prof. Baltazar is very saddened by the news, and decides to investigate the effect of the pollution of the river at his location.

From a sample taken from the river, he finds out that the diffusion constant of the fuel (in water) is $D = 0.04m^2/s$, and that the fuel stays at the surface (of the water). He estimates the river to be approximately $W = 10m$ wide, and assumes that the fuel spreads rapidly over this width. Moreover, since the current in the river is very low, he assumes that the main transport mechanism is *diffusion*. From this information, Prof. Baltazar predicts what will happen to the pollution in the river at his location.

You are asked to reproduce Prof. Baltazar's calculations by answer the following questions:

**a)** Let $C(x,t)$ denote the concentration of the fuel at position $x$ along the river at time $t$. Write down the equation satisfied by $C(x,t)$ and the corresponding initial condition.

**b)** Argue why the question of the fuel pollution "*reaching*" Prof. Baltazar is not well posed if no additional information is given.

One way to define "reaching" a given position $x$, is the time $t$ it takes for the (fuel) concentration (from the time of the spill) to reach a given level $c_0$, *i.e.* $C(x,t) = c_0$, for the first time, where, *e.g.*, the constant $c_0$ could be the lowest detectable concentration. Alternatively, it can be defined, as we will adopt in this exercise, as the time needed for the concentration profile to spread to a width so that $x - x_0 = \pm 2\sigma$, where $x_0$ is the location of the source, and $\sigma$ is the standard deviation of the diffusion process in our case.

**c)** How long does it take ($t = t_r$) for the fuel spill to reach Prof. Baltazar's cottage using the above definition of "reaching"? [Hint: How is $\sigma$ related to $D$?]

**d)** What is the fuel concentration, $C_r = C(x = L, t_r)$ at Prof. Baltazar's cottage at that time?

**e)** When does the fuel concentration reach its maximum at Prof. Baltazar's cottage? What is the concentration at this time?

**f)** After how long time has the concentration dropped to half of its maximum value?

**g)** The equation $C(x = L, t) = C_r$ has two solutions. What is the other solution to this equation? What is the interpretation of these two times?

**h)** Plot the concentration as a function of time. From this plot, do you find your answer from point c) reasonable?

**Some answers:**

c) $125,000s \simeq 35h$

d) $0.054g/m^2$

e) $t_m = L^2/2D = 5 \cdot 10^5 s \simeq 139h; \quad 0.121g/m^2$

g) $6,302,116s \simeq 1751h$
This is along time.... and gives some perspectives on the time-scale for pollution spreading!