7.14

$$ds^2 = -(1-Ar^2)^2 dt^2 + (1-Ar^2)^2 dr^2 + r^2 d\Omega$$

a. radial line $dt = d\varphi = dr = 0$

$$s = \int_0^R (1-Ar^2)\, dr = \left[r - \tfrac{1}{3}Ar^3\right]_0^R = R\left(1 - \tfrac{1}{3}AR^2\right)$$

b. $A = \int \sqrt{g_2}\, d\vartheta\, d\varphi$ $\qquad \sqrt{g_2} = R \cdot R \sin\vartheta$

$$= R^2 \int_0^\pi d\vartheta \int_0^{2\pi} d\varphi\, \sin\vartheta = 4\pi R^2$$

c. $V = \int \sqrt{g_3}\, dr\, d\varphi\, d\vartheta$ $\qquad \sqrt{g_3} = (1-Ar^2)\, r^2 \sin\vartheta$

$$= \int_0^R dr \int_0^\pi d\vartheta \int_0^{2\pi} d\varphi\; r^2 (1-Ar^2) \sin\vartheta = 4\pi \int_0^R dr\, (r^2 - Ar^4)$$

$$= \frac{4\pi R^3}{3}\left(1 - \frac{3}{5}\underbrace{AR^2}_{\xi^2}\right) \qquad \xi = \sqrt{A}\, R$$

d. $\sqrt{g} = (1-Ar^2)^2\, r^2 \sin\vartheta$

$$V_4 = \int_0^T dt \int_0^R dr \int_0^\pi d\vartheta \int_0^{2\pi} d\varphi\, (1-Ar^2)^2\, r^2 \sin\vartheta = 4\pi T \int_0^R dr\, \underbrace{(r - Ar^3)^2}_{r^2 - 2Ar^4 + A^2 r^6}$$

$$= \frac{4\pi}{3} R^3 T \cdot \left\{1 - \frac{6}{5}\xi^2 + \frac{3}{7}\xi^4\right\} \qquad \frac{R^3}{3} - \frac{2}{5}AR^5 + \frac{1}{7}A^2R^7$$