General-relativistic hydrodynamics around single and binary black holes

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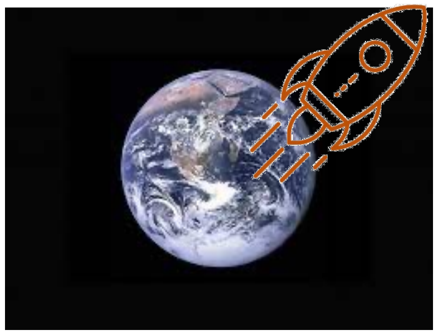
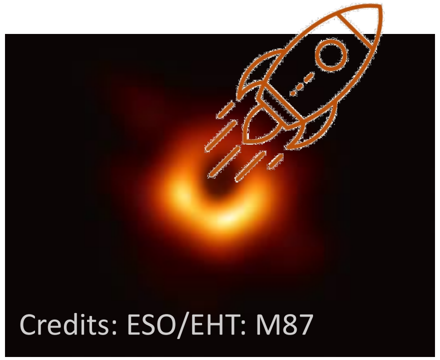
Things you may ignore about black holes

1. They were conjectured BEFORE general relativity (XVIIIe century)



$v > v_{\mathrm{esc}} \sim 10\mathrm{km/s}$



$v > v_{\mathrm{esc}} > c$

2. They are not defined by their density but by their compactness
   $$\Xi \equiv \frac{GM}{c^2 R} = \frac{R_s}{R} \propto M/R$$

3. They are among the simplest astrophysical objects: mass, spin, charge
   (the so-called « No-hair theorem »)

4. We still do not know how they all form

5. We believe each galaxy hosts a supermassive black hole at their center

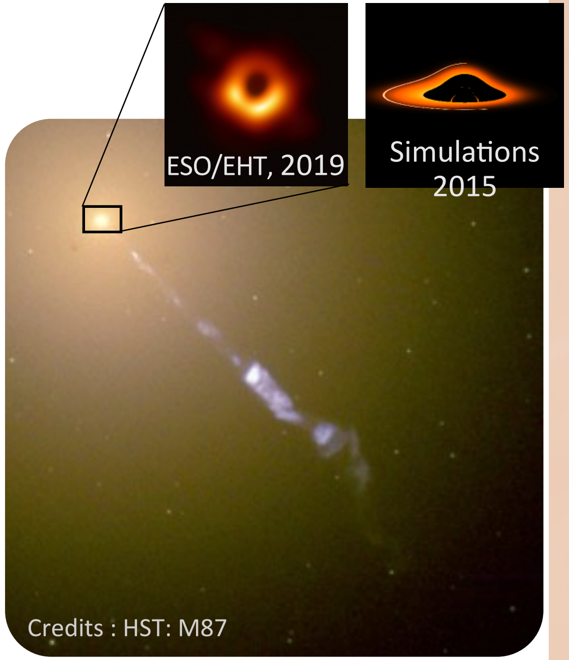
Some galaxies are boring

What we believe the Milky Way looks like

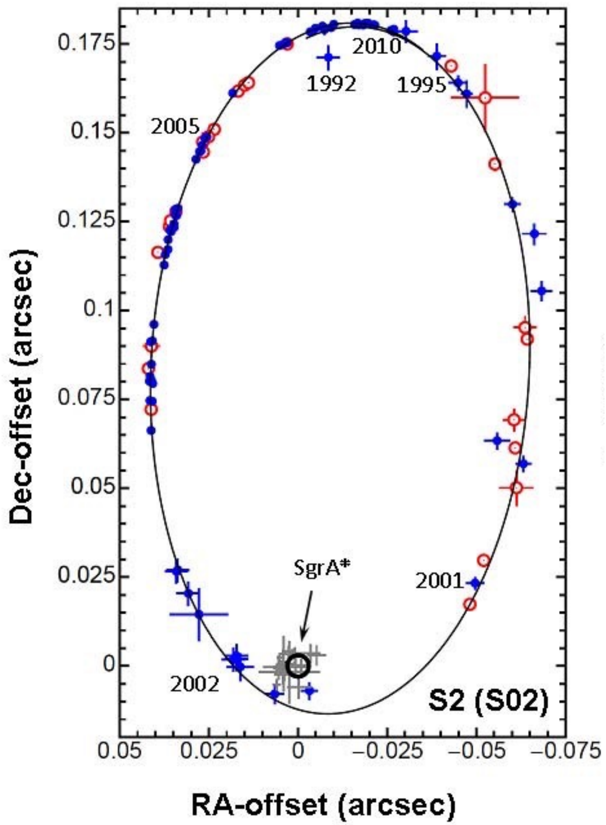
... and some are really not

The « antennae » galaxies about to merge
(This is us, in 4Gyr, with Andromeda)

An « active galactic nucleus » (AGN)



Zooming-in on Sagittarius A*, our Galactic center



Genzel+10

Milky Way: Sagittarius A*, indirect evidence of a black hole

How ?

Measure star's velocity + apply Kepler's 3rd law:

$$P = 2\pi\sqrt{\frac{r^3}{G\,M_{\rm tot}}}$$

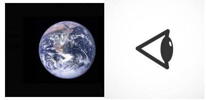
$$\rightarrow M_{\rm tot} \sim 10^6 \mathrm{M}_\odot$$

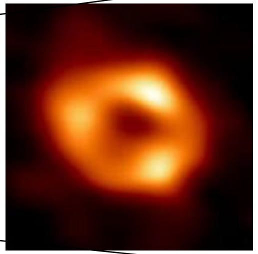
$M_{\rm Sgr} \sim M_{\rm tot}$ in a radius $< r \rightarrow$ compactness $\Xi \equiv \frac{GM_{\rm Sgr}}{c^2 r} \ll 1$!

This suggests the presence of a supermassive black hole

Zooming-in on Sagittarius A*, our Galactic center

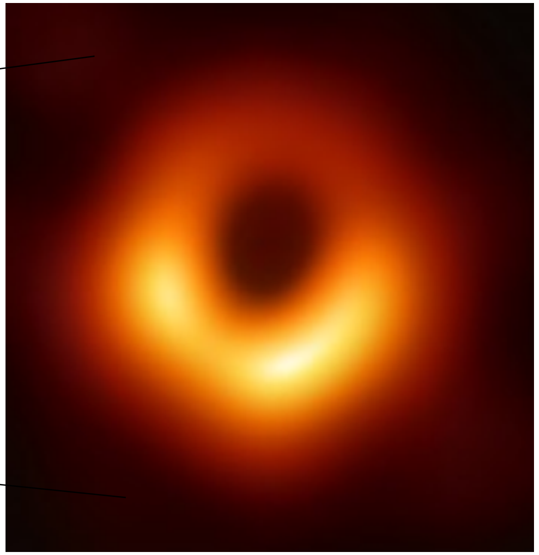
radio emission, likely synchrotron, detected with the Event Horizon Telescope (Nobel Prize 2020)





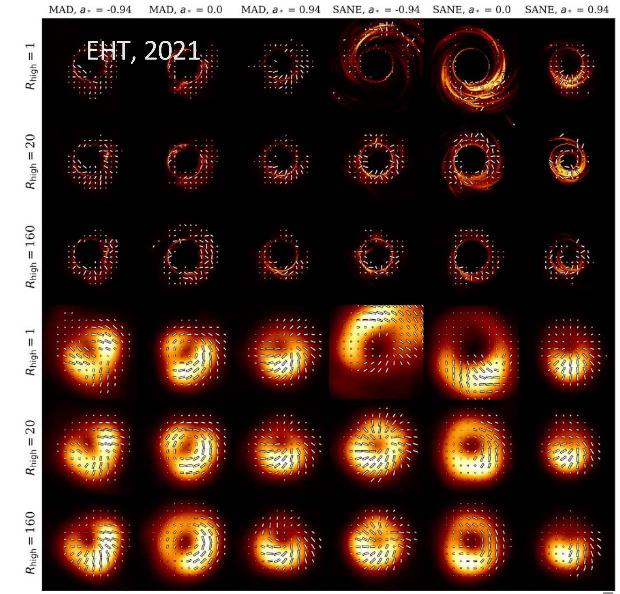
Sagittarius A*
(Milky Way)

x1000



M87
Galaxy

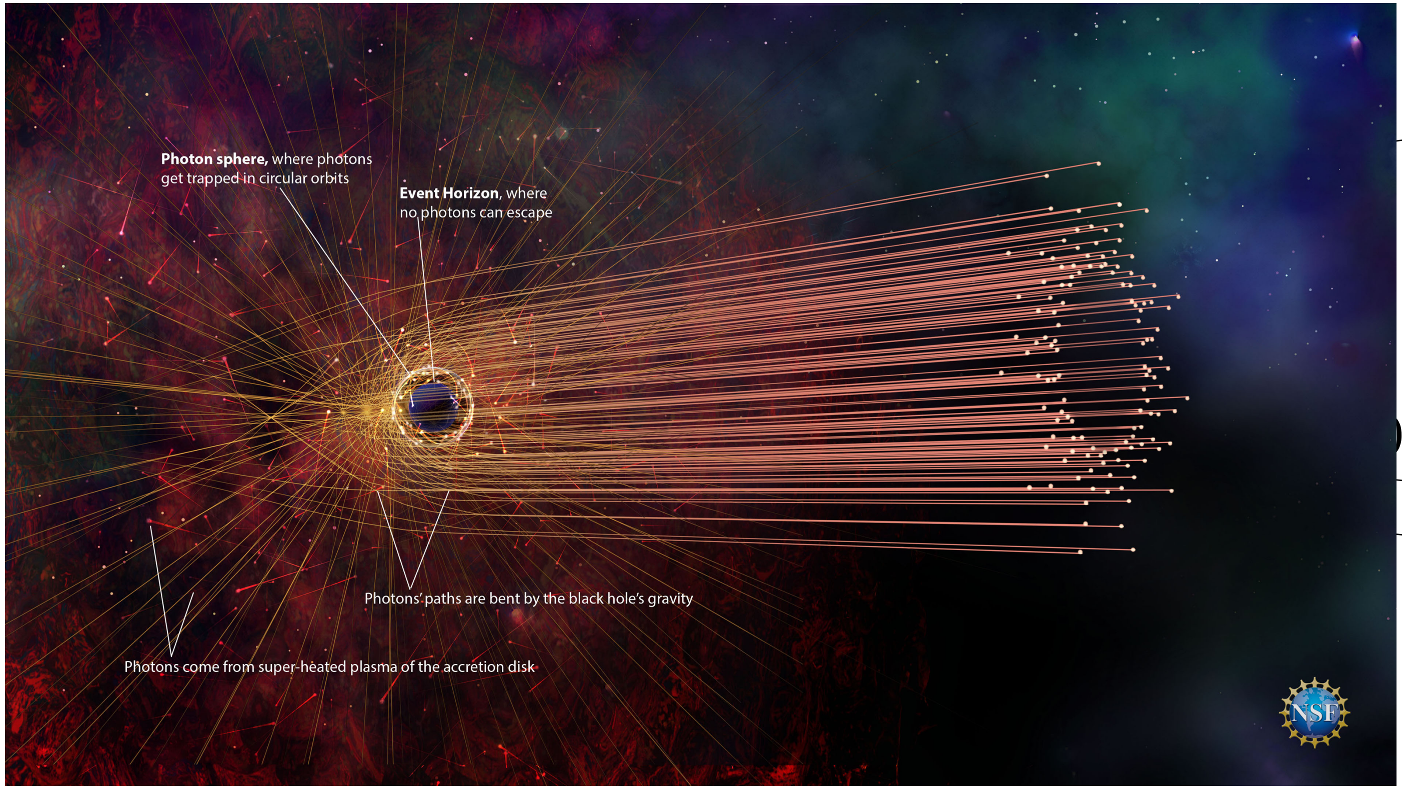
R_S



But what do we see exactly? at these frequencies, the photon ring
What are the properties of the surrounding plasma?
What is the inclination of the accretion disk plane wrt line-of-sight?
What is the black hole spin?

➢ Need to solve the equations of hydrodynamics around a black hole: General-relativistic hydrodynamics

Zooming-in on Sagittarius A*, our Galactic center



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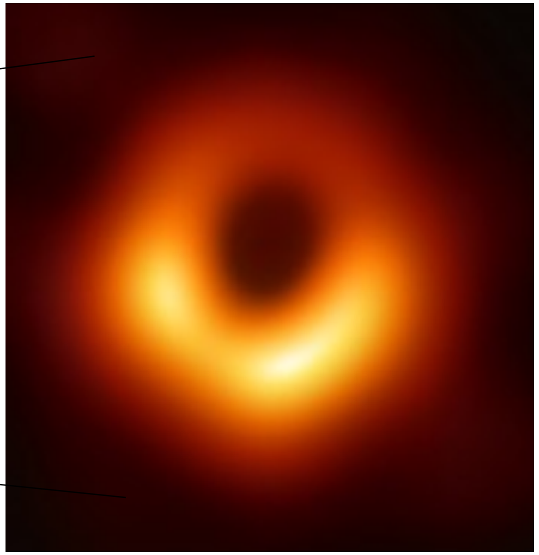
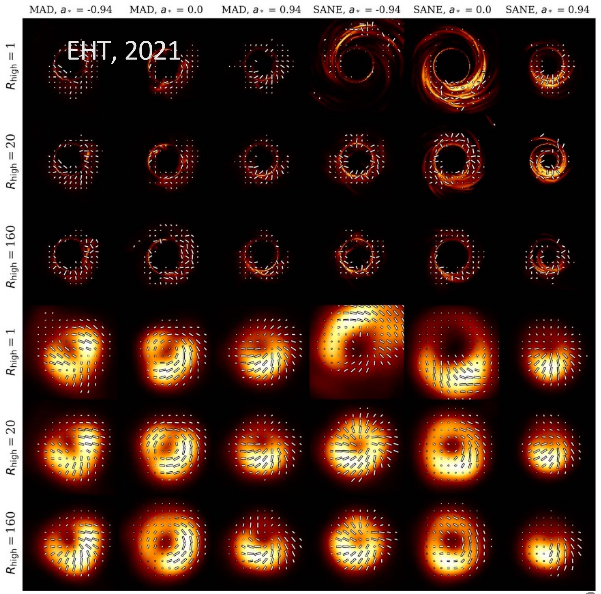


M87
Galaxy



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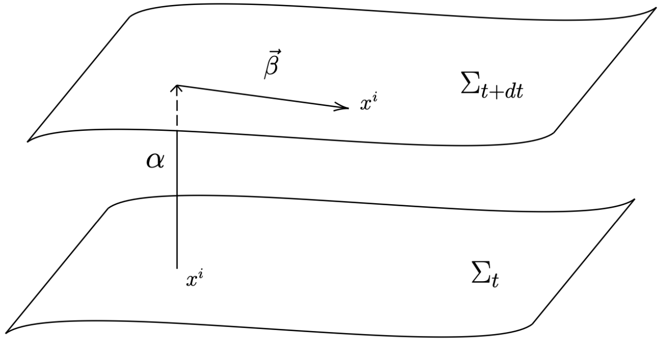
➢ Need to solve the equations of hydrodynamics around a black hole: General-relativistic hydrodynamics

Fluid simulations : 3+1 spacetime decomposition

- **3+1 decomposition :** spacetime foliated into non-intersecting spacelike hypersurfaces

- All metrics can be decomposed into the $\alpha, \beta_i, \gamma_{ij}$:

$$g_{\mu\nu} = \begin{matrix} \beta^2 - \alpha^2 & \beta_i \\ \beta_i & \gamma_{ij} \end{matrix}$$



Credits:
Daniel Heinesen

for comparison:

Minkowski (flat spacetime) in Cartesian coordinates :

$$\eta_{\mu\nu} = \begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Kerr (rotating black hole) in Boyer-Lindquist coordinates - p.74 of your GR lecture notes :

$$g_{\mu\nu} = \begin{matrix} 1 + \frac{2Mr}{\rho^2} & 0 & 0 & \frac{4Mar\sin^2\theta}{\rho^2} \\ 0 & -\rho^2/\Delta & 0 & 0 \\ 0 & 0 & -\rho^2 & 0 \\ \frac{4Mar\sin^2\theta}{\rho^2} & 0 & 0 & -(r^2 + a^2 + \frac{2Ma^2r\sin^2\theta}{\rho^2})\sin^2\theta \end{matrix}$$

The presence of the mixed term $g_{t\phi}$ means that infalling particles (and thus space-time) is dragged around the rotating black hole.

Fluid simulations : 3+1 spacetime decomposition

- **Local conservation of the stress-energy tensor and matter current density:**

Relativistic density and momentum

Lapse scalar

Shift vector

Mass conservation ⇒

$$\partial_t(\mathcal{D}) + \partial_j\left[\mathcal{D}\left(\alpha \mathrm{v}^j - \beta^j\right)\right] = 0$$

Momentum conservation ⇒

$$\partial_t(\mathcal{S}_i) + \partial_j\left(\left[\mathcal{S}_i(\alpha \mathrm{v}^j - \beta^j) + \alpha\mathcal{P}\delta^j_i\right]\right) =$$

$$-(W\mathcal{D} + \mathcal{P}(W^2 - 1))\partial_i\alpha + \frac{\alpha}{2}\left(\mathcal{S}^j\mathrm{v}^k + \mathcal{P}\gamma^{jk}\right)\partial_i\gamma_{jk} + \mathcal{S}_j\partial_i\beta^j$$

Spatial metric

- No back-reaction of the fluid onto the metric (i.e. no self-gravity)

- Back-reaction of the fluid's energy and mass: Numerical Relativity

Einstein equation :

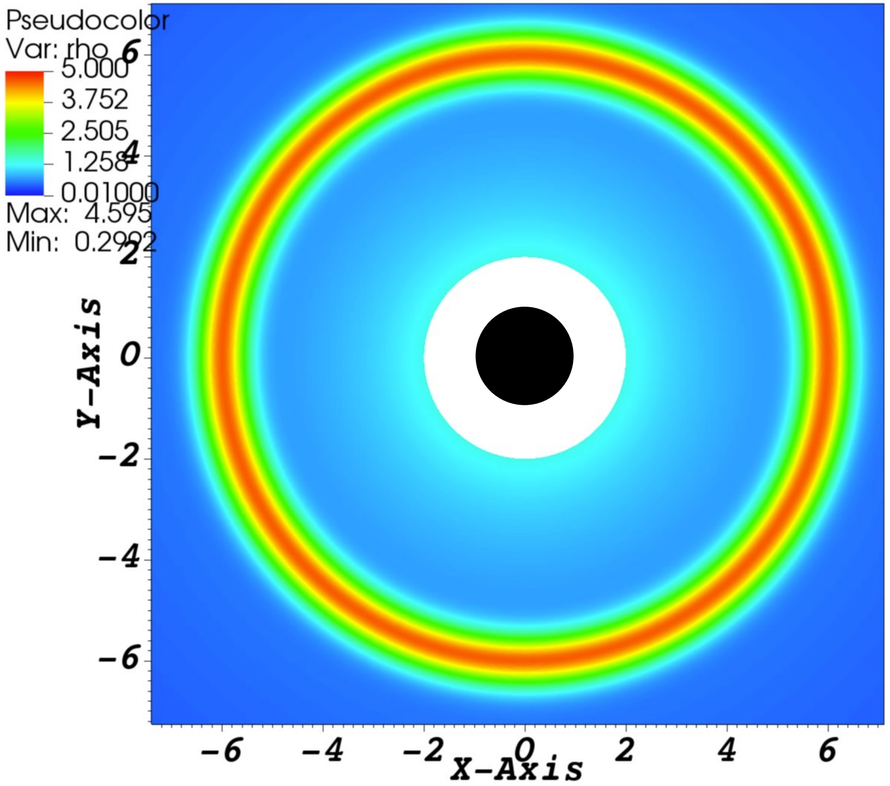
$$G_{\mu\nu}[g_{\alpha\beta}] = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Determine space time structure

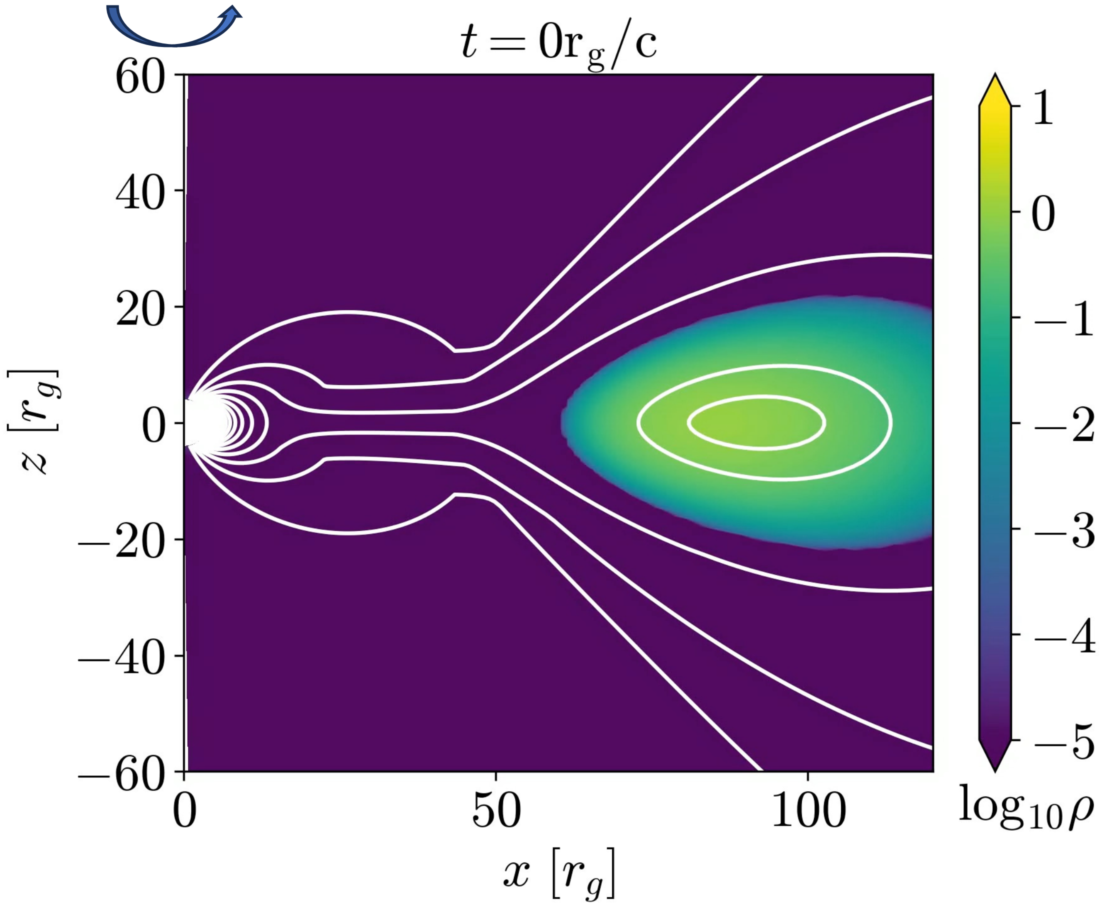
Source of gravity (e.g. moving star)

Examples

Black hole



Neutron star

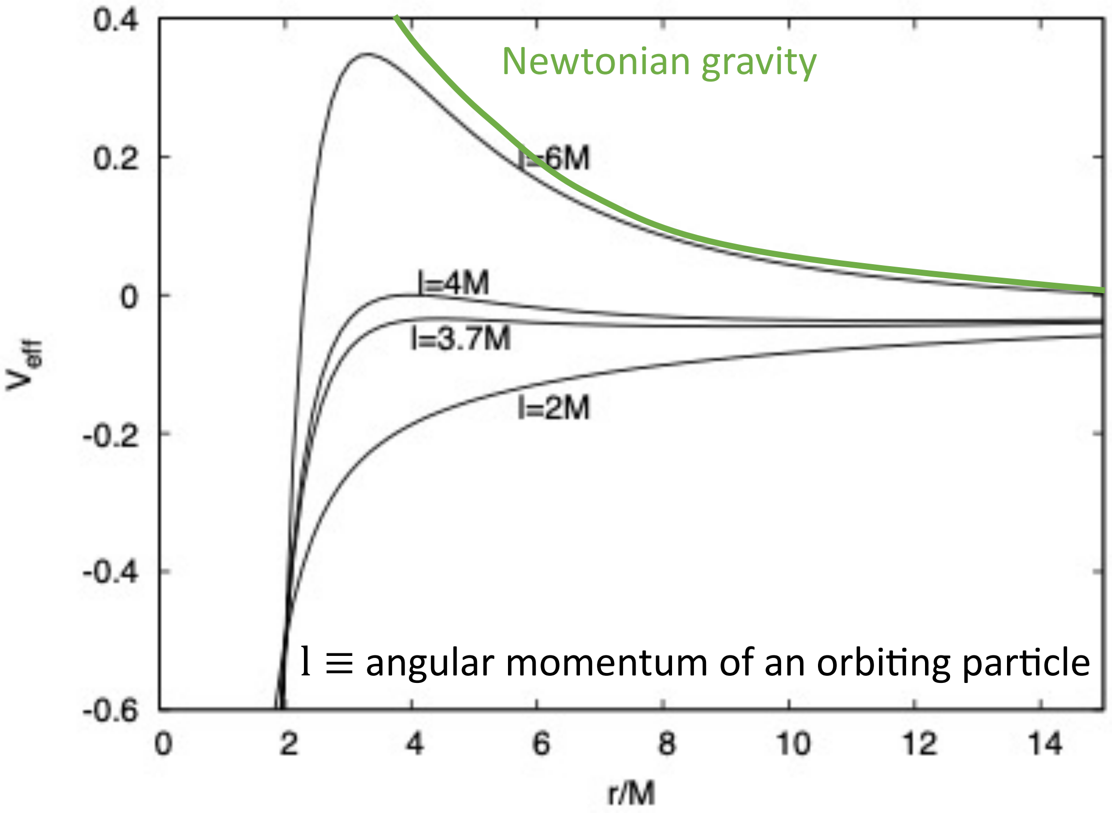


What for ?

- Explain the observational properties of black holes and neutron stars: e.g. fast variability
- Understand how matter behaves in strongly curved spacetimes
- Looking for indirect clues of exotic compact objects (e.g. boson stars)
- …

Example of a direct effect from GR

- Around a Schwarzschild black hole exists a so-called « innermost stable circular orbit » (ISCO) – Fig. 4.1 of your lecture notes



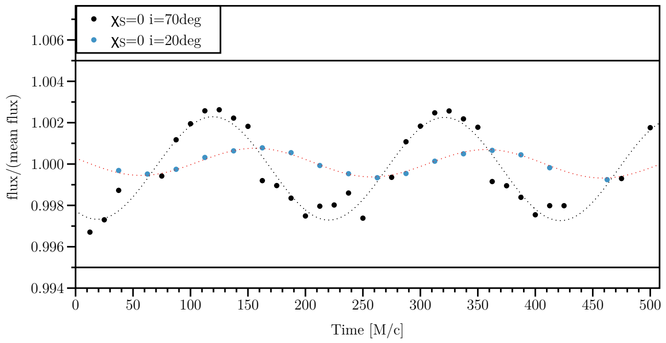
l ↗ ⇒ centrifugal force (outward) ↗

➢ An accretion disk should be truncated at this ISCO

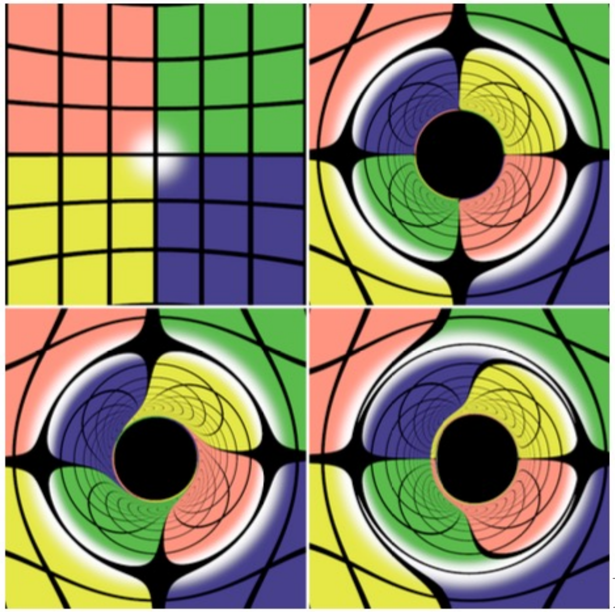
Why using a GR ray-tracing code ?

- **Concept: solve the geodesic equation for photons back from the observer (Earth) to the source**

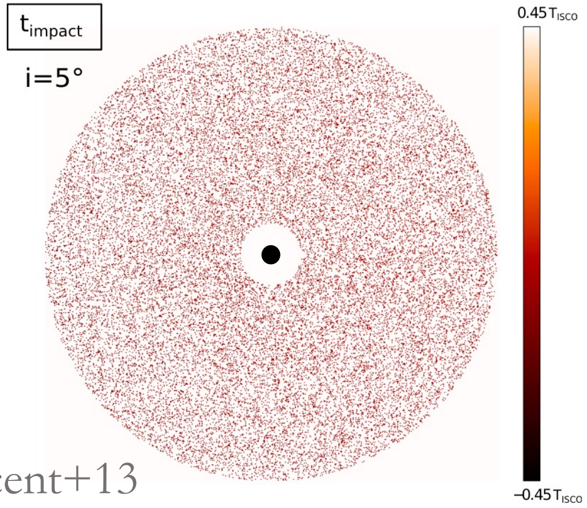
- **Relativistic ray-tracing:**
  e.g. Doppler beaming: matter approaching the observer appears brighter
  → an orbiting dense blob produces a sinusoid in the luminosity



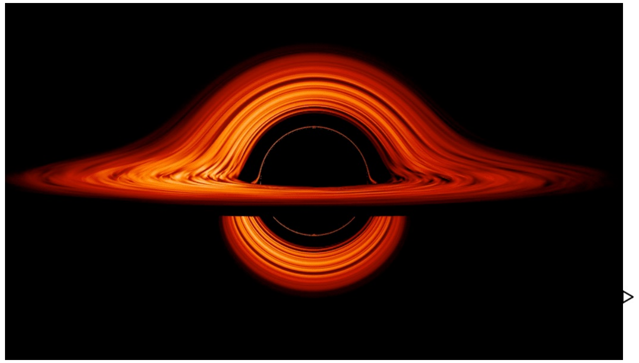
- **GR** effects:
  Light deflection (p. 57)
  « Shapiro effect »: time delay



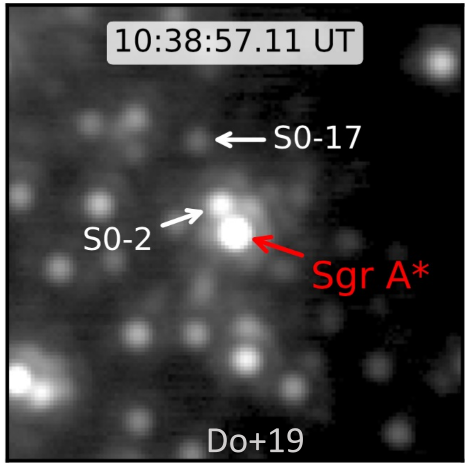
Bohn+15

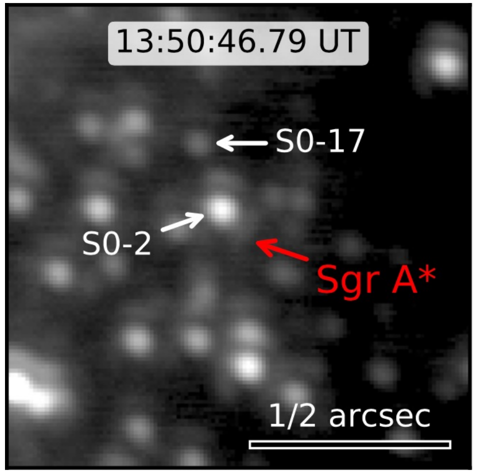


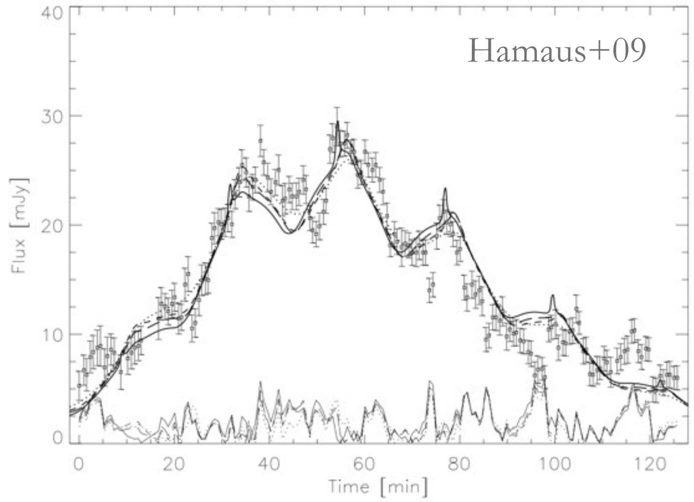
Vincent+13

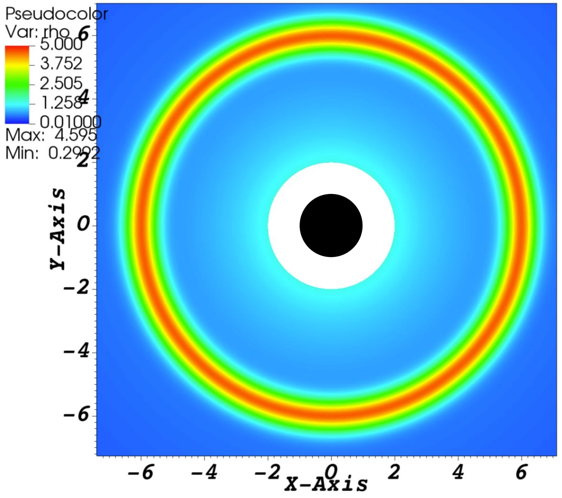


Example of application: origin of the eruptions of Sgr A*

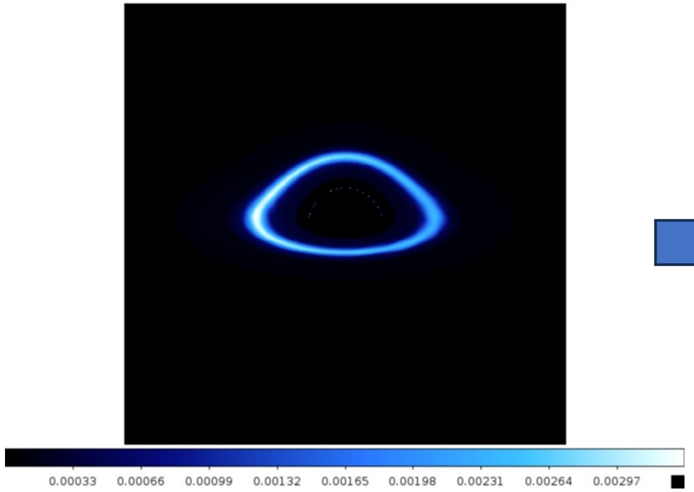




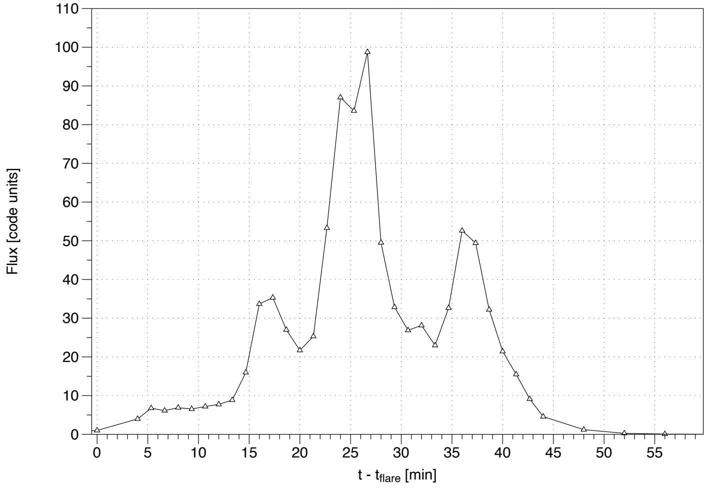




Model a magneto-hydrodynamical instability around a black hole



Synchrotron emission map (i.e. interaction of electrons with magnetic field)



Synthetic lightcurve

→ A repeating instability in the accretion disk could be at the origin of the eruptions of Sgr A*

This was not the first direct evidence of the existence of black holes.

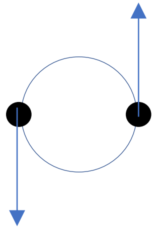
What was ?

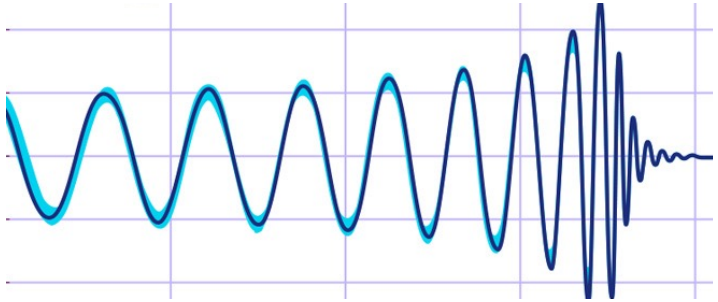
A new window on the Universe : gravitational waves

Einstein's equations linearized in a flat, perturbed spacetime:

$$\Box h^{\mu\nu} = \left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right) h^{\mu\nu} = 0$$

Wave equation

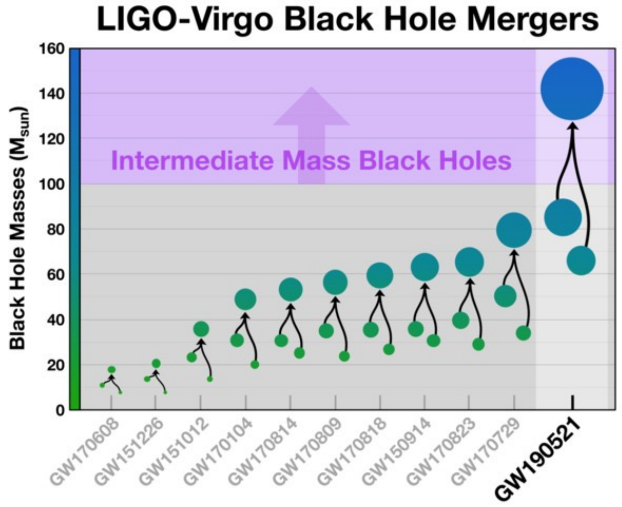




Stellar-mass BHs

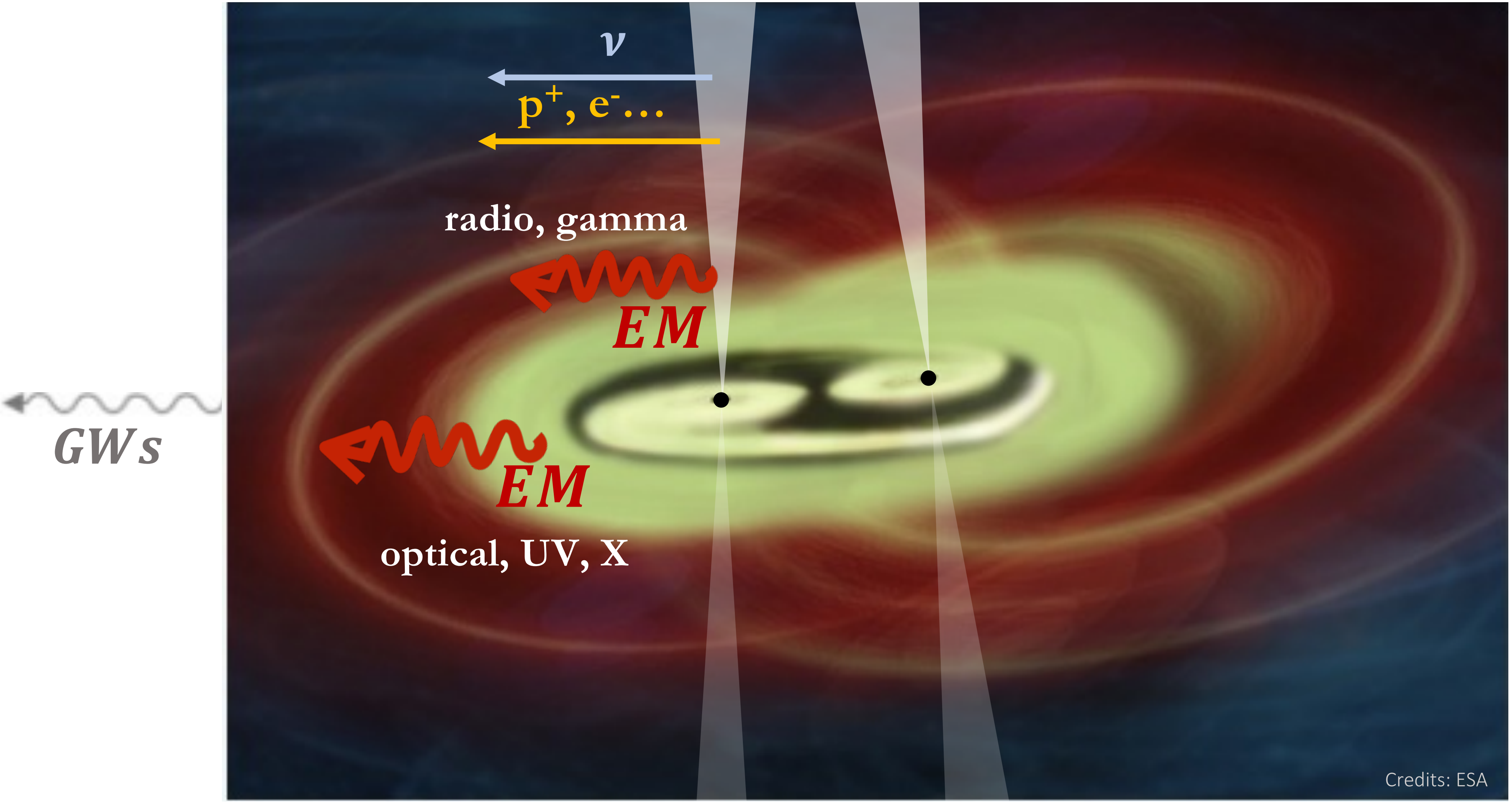
~1kHz

LIGO/Virgo



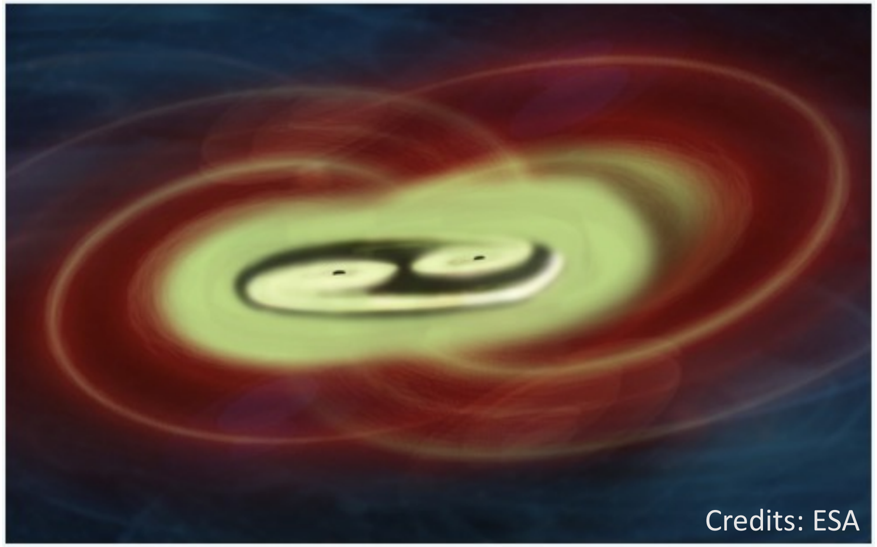
First direct detection of BHs

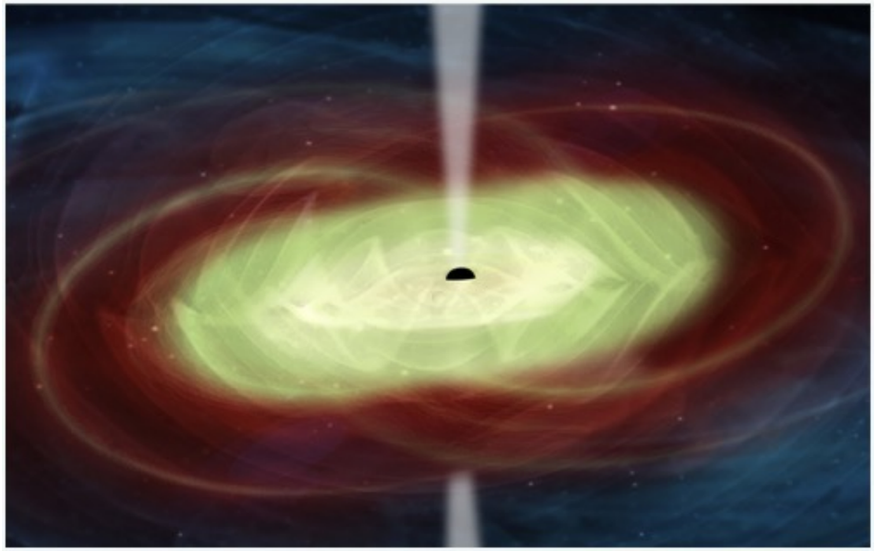
Accreting binary black holes: multi-messenger sources



Electromagnetic counterpart to binary black hole merger

Need a gas-rich environment:
X Stellar-mass black holes (LIGO/Virgo sources)
✓ Supermassive binary black holes!
e.g. galaxy merger





- Binary black holes and their coalescence
  - Galaxy + black hole growth
  - Cosmology: Hubble constant
  - Fondamental physics: speed of gravity
  - Formation of active galactic nuclei?

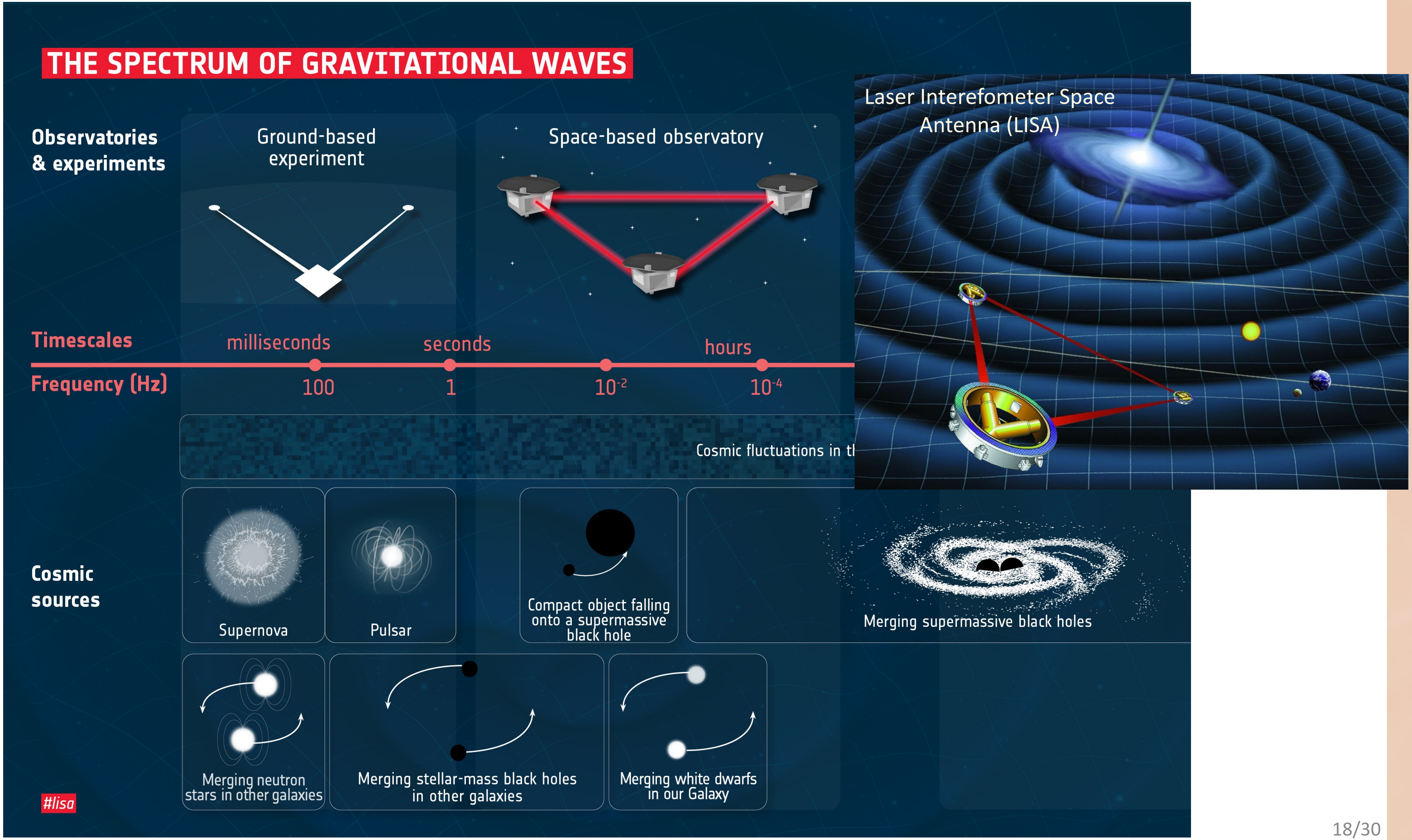
How to detect gravitational waves from supermassive binary black holes?

$f_{\text{GW}} \sim f_{\text{Kepl.}} \propto \sqrt{\frac{GM}{r^3}} \propto M^{-1}$ → Supermassive BBHs emit GW at smaller frequencies than LIGO/Virgo sources

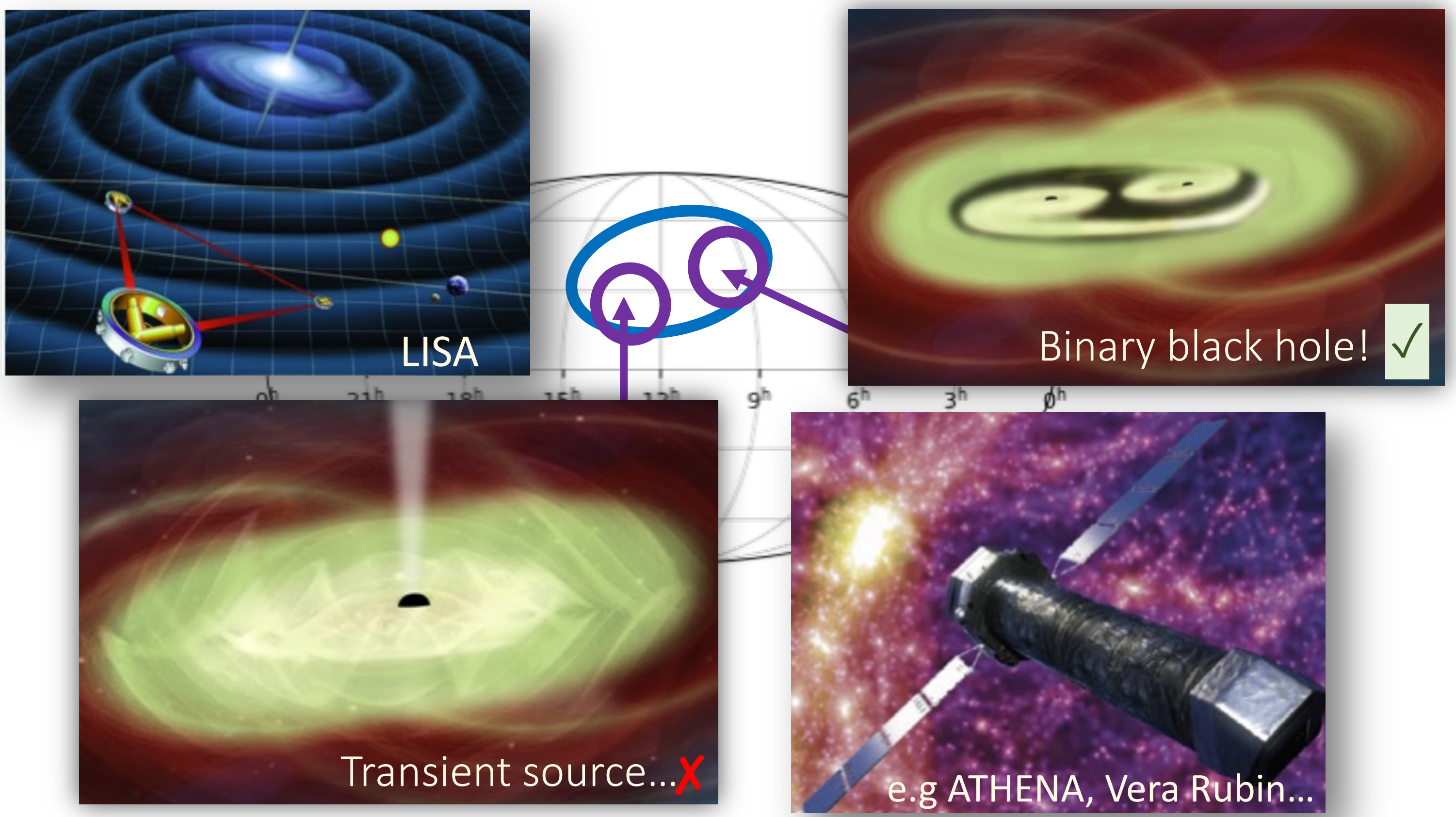
to be in units of $r_g \propto M$

$\lambda_{\text{GW}} \sim \frac{c}{f_{\text{GW}}} \propto M$ → Need a bigger experiment than LIGO/Virgo (~km)
How much bigger ?

Supermassive black holes have masses $\sim 10^{5-10} M_\odot$, i.e. 10^{4-9} times that of stellar-mass black holes → Need interferometers arms $> 10^4$ km long
Problem : 10^4 km $> R_{\text{Earth}}$


THE SPECTRUM OF GRAVITATIONAL WAVES
Observatories & experiments
Ground-based experiment
Space-based observatory
Timescales
milliseconds
seconds
hours
Frequency (Hz)
100
1
10-2
10-4
Cosmic fluctuations in th
Cosmic sources
Supernova
Pulsar
Compact object falling onto a supermassive black hole
Merging supermassive black holes
Merging neutron stars in other galaxies
Merging stellar-mass black holes in other galaxies
Merging white dwarfs in our Galaxy
#lisa
Laser Interefometer Space Antenna (LISA)

Electromagnetic follow-up after a LISA detection

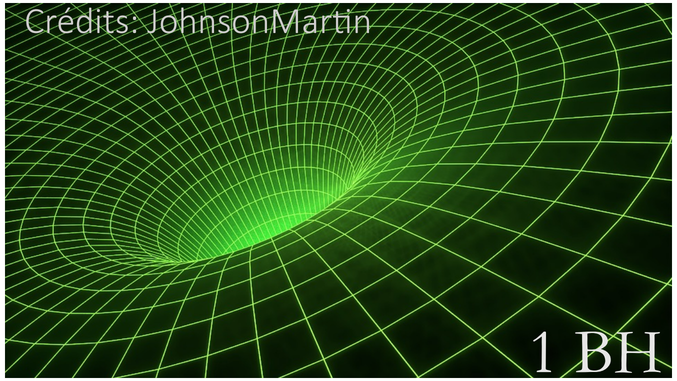


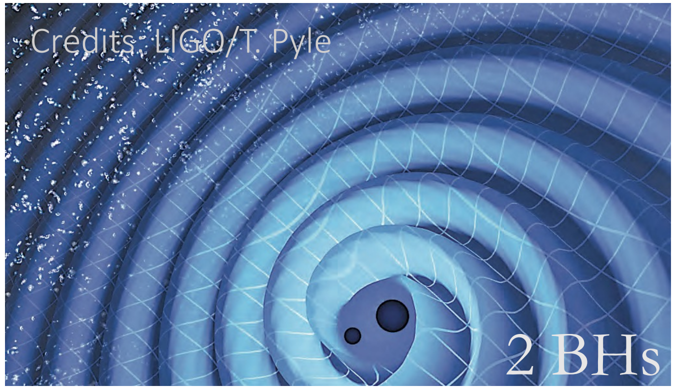
- LISA: space-based gravitational wave detector
  0.1-100 mHz band
  - SMBBH up to merger
  - Stellar-mass BH in early pre-merger stage only

- PTA: Pulsar Timing Arrays
  1nHz-100nHz band
  - Close individual SMBBH mergers

How to distinguish binary black holes from other (transient) sources ?

From single to binary black holes





$$g_{\mu\nu} = \begin{pmatrix} 1+\frac{2Mr}{\rho^2} & 0 & 0 & \frac{4Mar\sin^2\theta}{\rho^2} \\ 0 & -\rho^2/\Delta & 0 & 0 \\ 0 & 0 & -\rho^2 & 0 \\ \frac{4Mar\sin^2\theta}{\rho^2} & 0 & 0 & -(r^2+a^2+\frac{2Ma^2r\sin^2\theta}{\rho^2})\sin^2\theta \end{pmatrix}$$

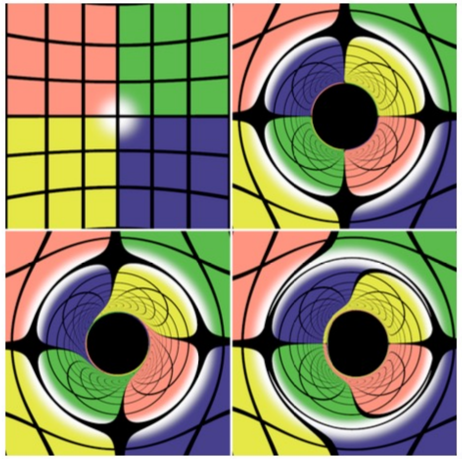
~~Stationarity~~

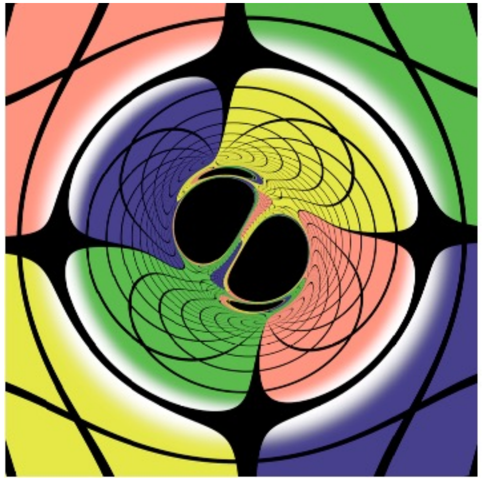
Delayed gravity

~~Axisymmetry~~

$$g_{\mu\nu} = \begin{pmatrix} g_{tt} & g_{tr} & g_{t\theta} & g_{t\phi} \\ g_{rt} & g_{rr} & g_{r\theta} & g_{r\phi} \\ g_{\theta t} & g_{\theta r} & g_{\theta\theta} & g_{\theta\phi} \\ g_{\phi t} & g_{\phi r} & g_{\phi\theta} & g_{\phi\phi} \end{pmatrix}$$

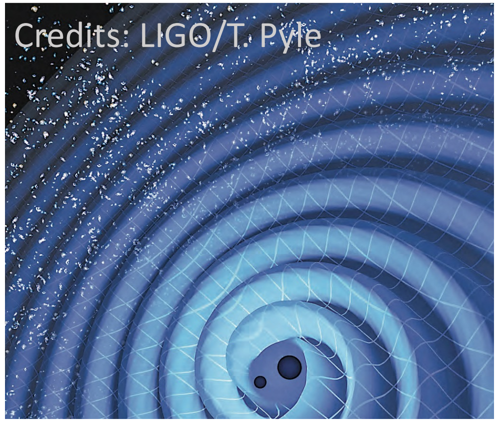
Bohn+15



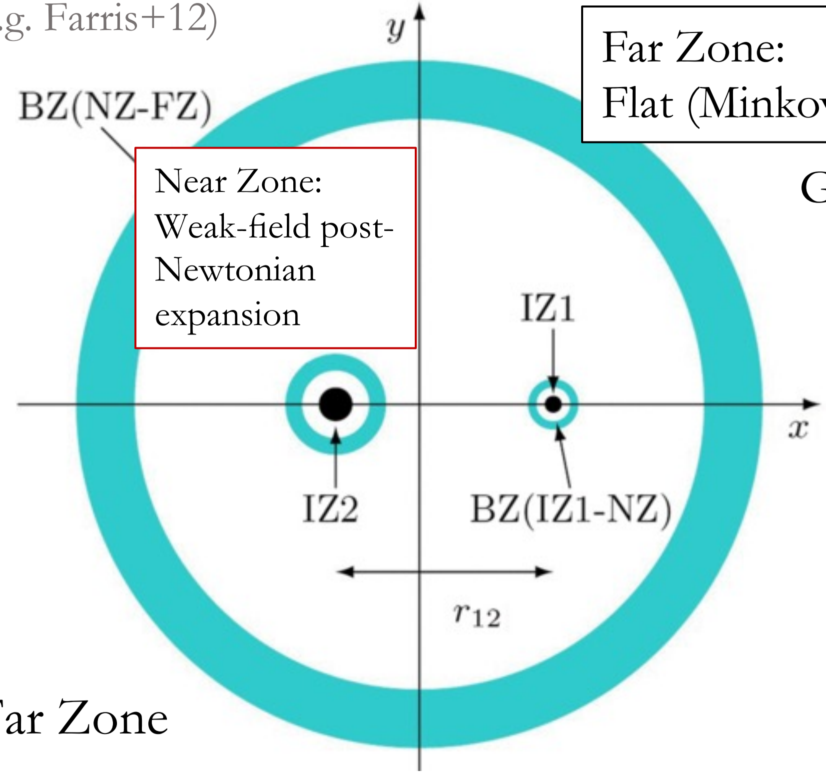


An approximate binary black hole spacetime

- Why not using Newtonian gravity ? (e.g. D'Orazio+13)
  GR IS important !!
- Why not solving the Einstein's equations ?
  Too expensive for >10 orbits simulations (e.g. Farris+12)



(Johnson-Mcdaniel+09)



Far Zone:
Flat (Minkowski) + outgoing GWs

GWs ⤳ accretion disk ?
MR+22, MNRAS
MR+23, A.N.

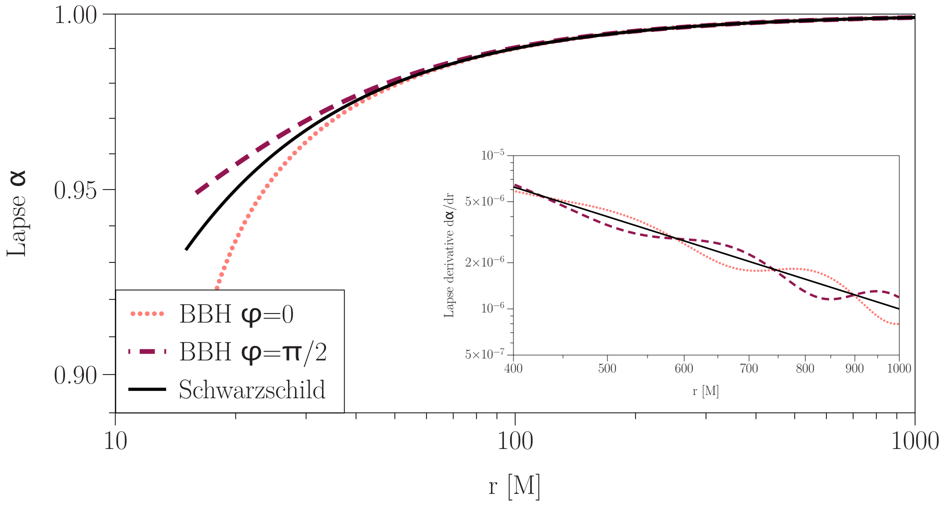
- A computationally-heavy construction: example Far Zone

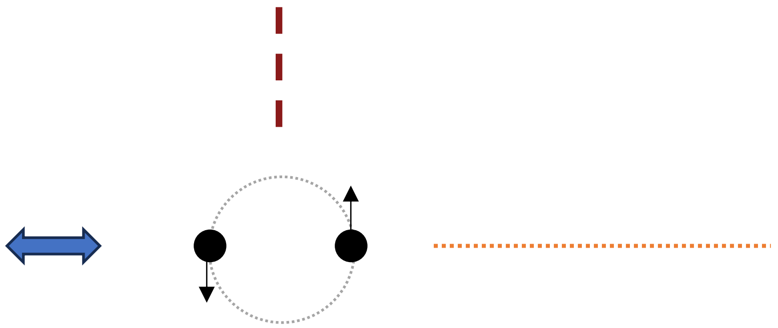
Ireland+16

$$g_{00}+1=\frac{2m_1}{r}+\frac{m_1}{r}\left\{v_1^2-\frac{m_2}{b}+2\left(\vec{v}_1\cdot\hat{n}\right)^2-\frac{2m}{r}+6\frac{\left(\vec{x}_1\cdot\hat{n}\right)}{r}\left(\vec{v}_1\cdot\hat{n}\right)-\frac{x_1^2}{r^2}+\frac{\left(\vec{x}_1\cdot\hat{n}\right)^2}{r^2}\left(3-2r^2\omega^2\right)\right\}$$
$$+\left(1\leftrightarrow 2\right)+O(v^5),$$

- Construction valid down to $r_{12}\sim 8M$ (because $v > 0.1$ c, slow-motion approx. for PN breaks down)

What does a binary black hole metric look like?



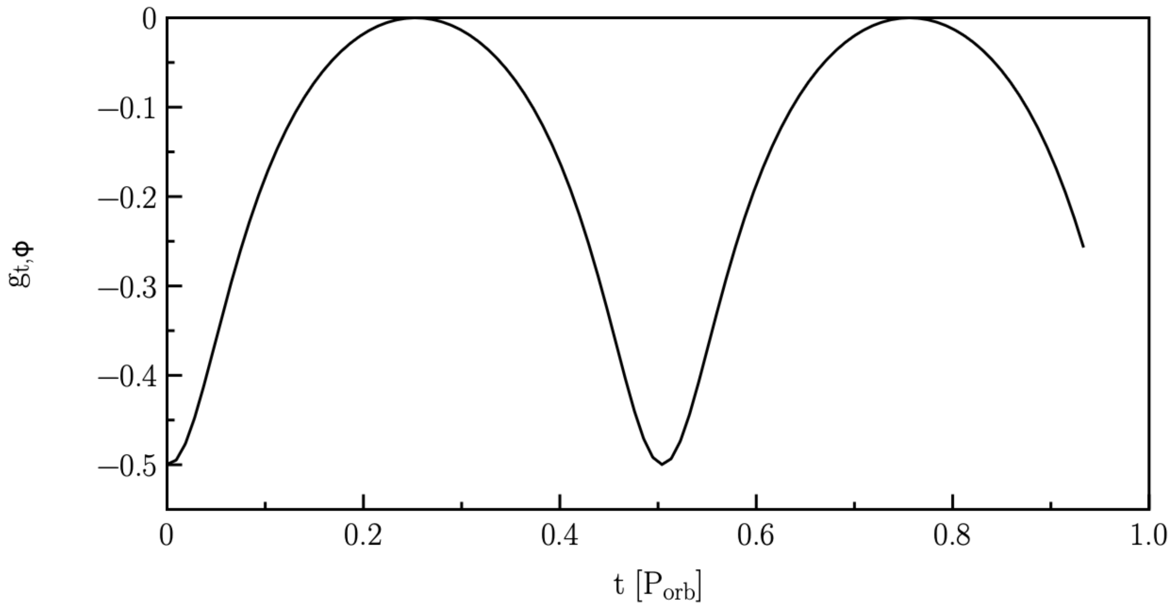


- Far from the binary, similar to a single BH, except for GW residual

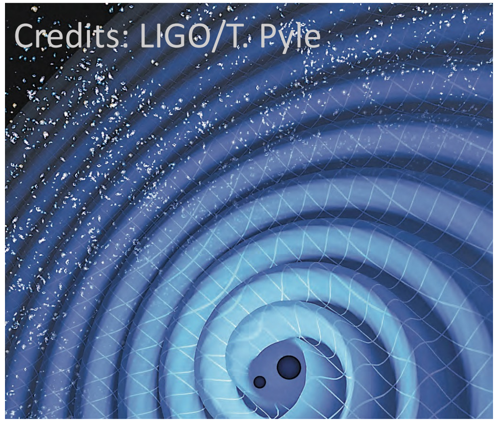
$g_{t\phi} \neq 0 \rightarrow$?

$$
g_{\mu\nu} = \begin{pmatrix} 1+\frac{2Mr}{\rho^2} & 0 & 0 & \frac{4Mar\sin^2\theta}{\rho^2} \\ 0 & -\rho^2/\Delta & 0 & 0 \\ 0 & 0 & -\rho^2 & 0 \\ \frac{4Mar\sin^2\theta}{\rho^2} & 0 & 0 & -(r^2+a^2+\frac{2Ma^2r\sin^2\theta}{\rho^2})\sin^2\theta \end{pmatrix}
$$

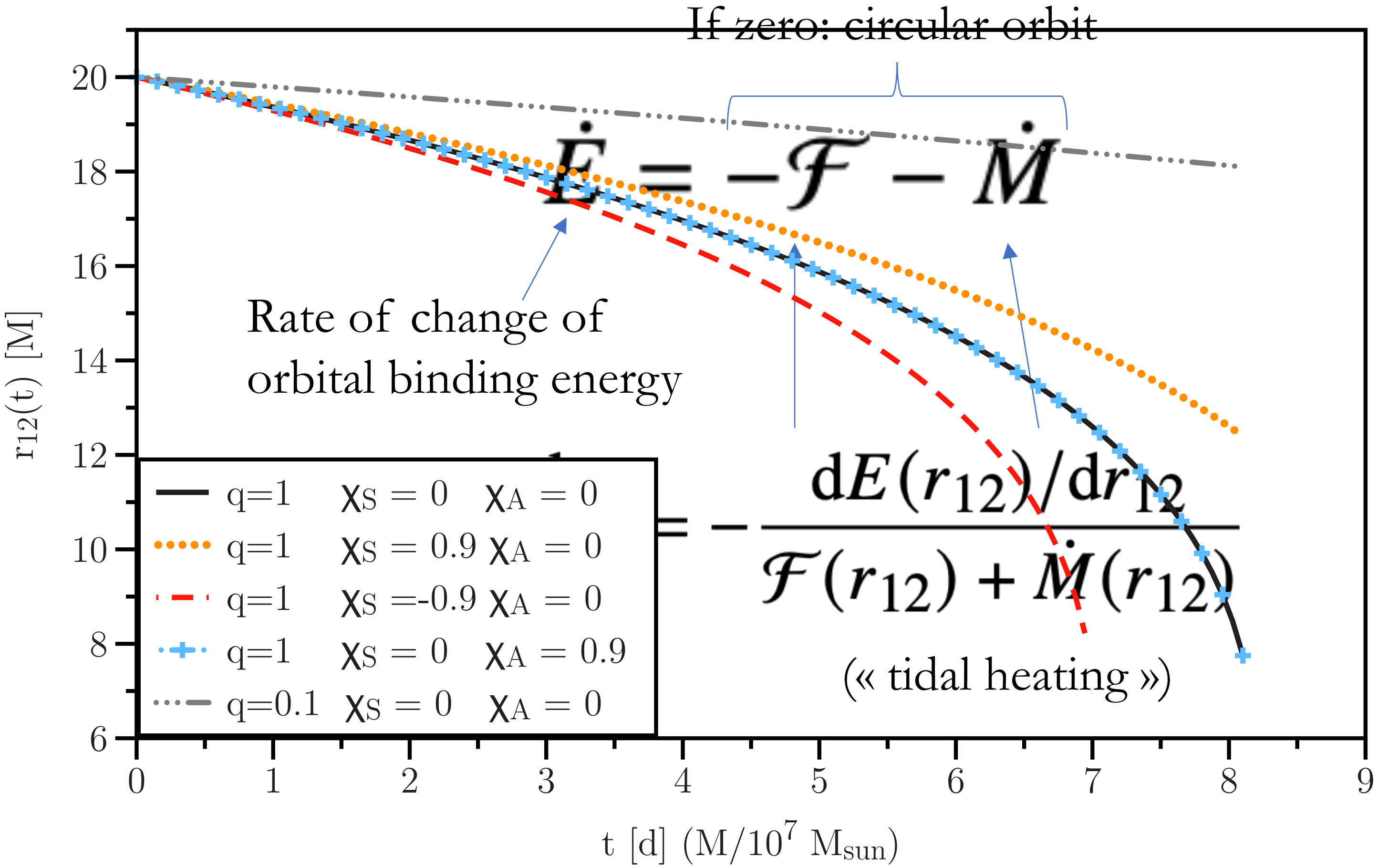
- Frame-dragging (Lense-Thirring) effect, as in the Kerr metric, but due to the orbital motion of the BBH



Inspiral equation of motion



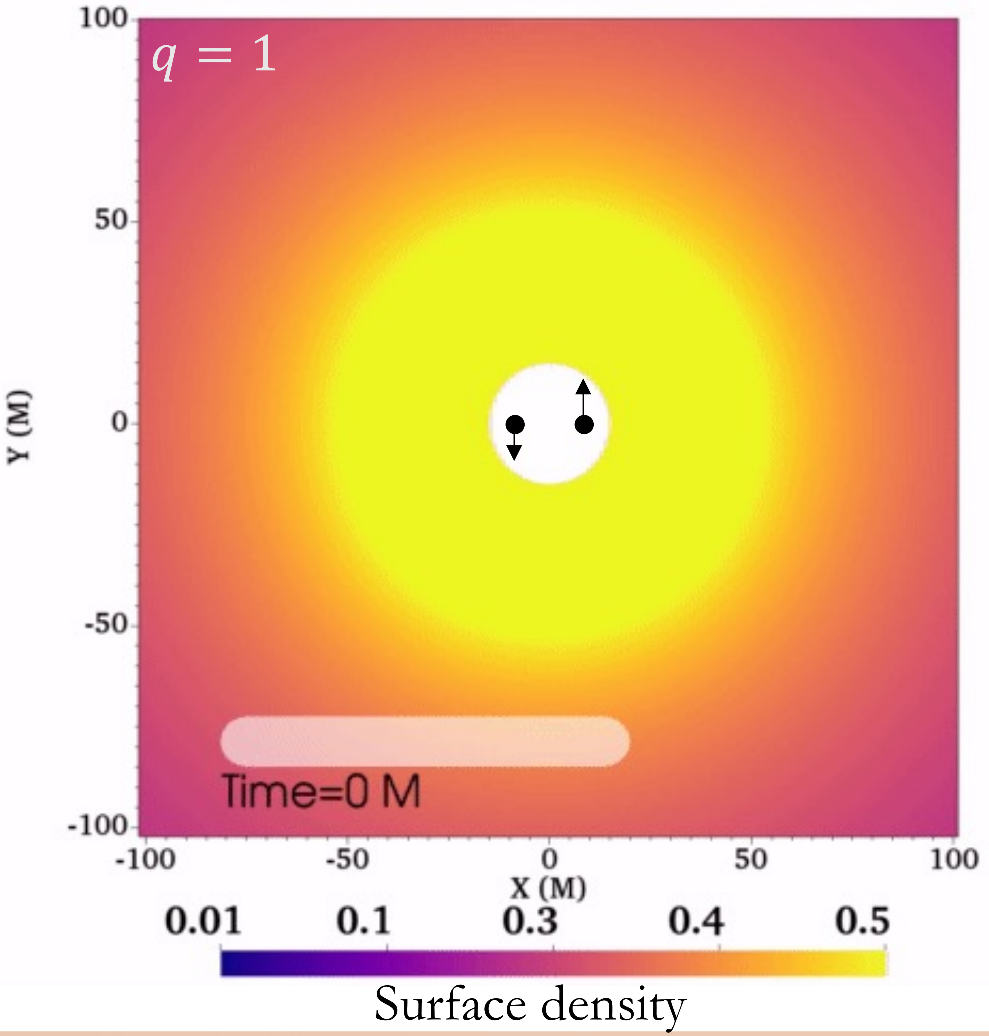
- ✓ 3.5 Post-Newtonian inspiral motion for **orb. separation and orb. frequency**
- ✓ **valid for spinning BBHs**



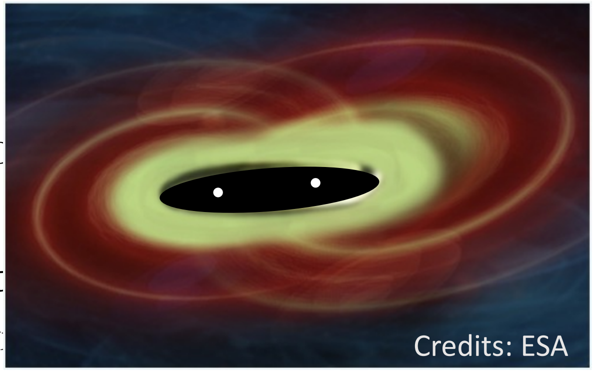
- **Recover the orbital hang-up effect**
- **Slower inspiral for *q* ↘**

Fluid simulations: accretion structures

- 2D General-relativistic-hydrodynamical simulations of a circumbinary disk
- BBH approximate metric (Mundim+14, Ireland+16)
- Excised inner region



In circular orbit, fo



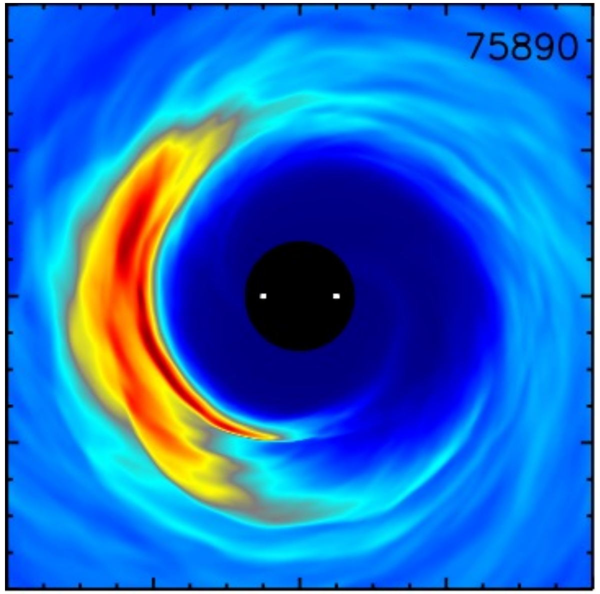
1. A cavity at
   (Artymow
2. Streams (Artymowicz+96) & spiral arms

   and further in time…

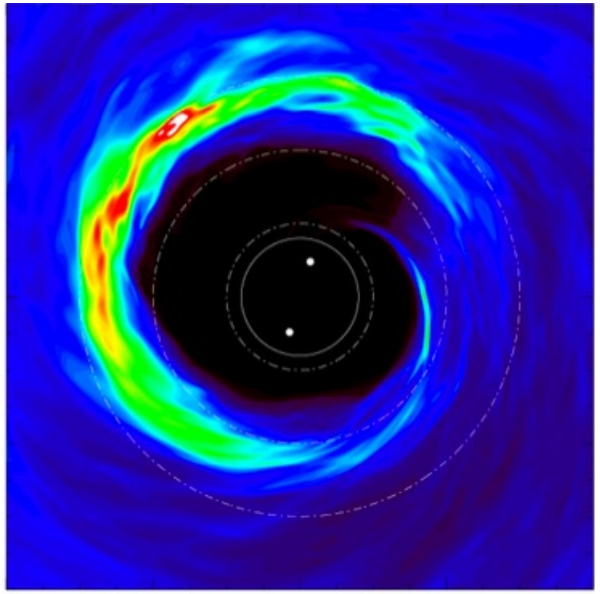
3. An overdensity, or « lump »
   (e.g. MacFadyen+08, Shi+12, Noble+12, D'Orazio+13, Gold+14, Farris+14, Ragusa+16, Miranda+17, Muñoz+19, Duffell+20, Armengol+21, Tiede+20+21, Liu+21, Franchini+22 (priv. com.), Siwek+22, Cimerman+23…)

Accretion structures → Observational features?

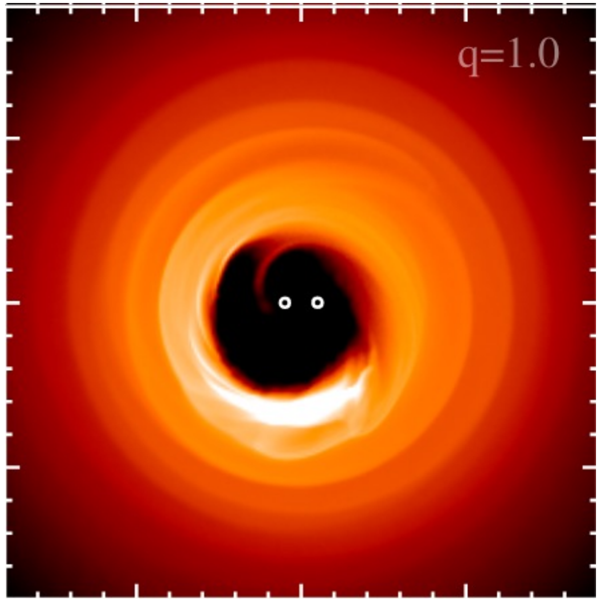
The need for complementary studies in computational astrophysics



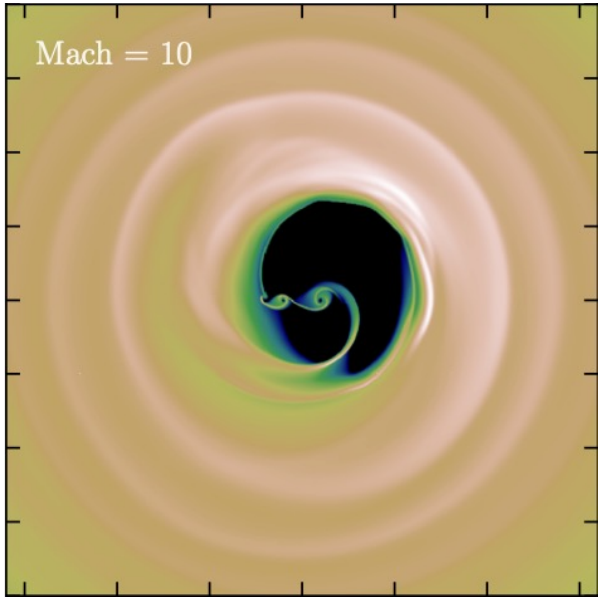
Noble+12, 3D GRMHD



Shi+12, 3D MHD



Ragusa+20, 3D SPH

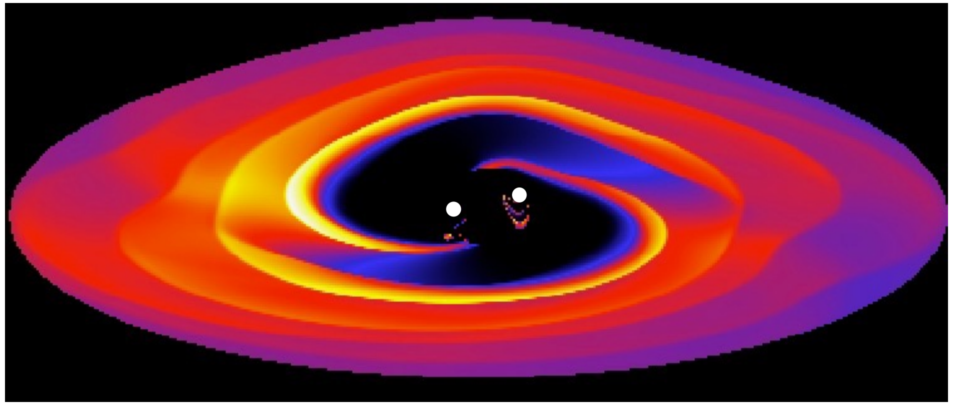


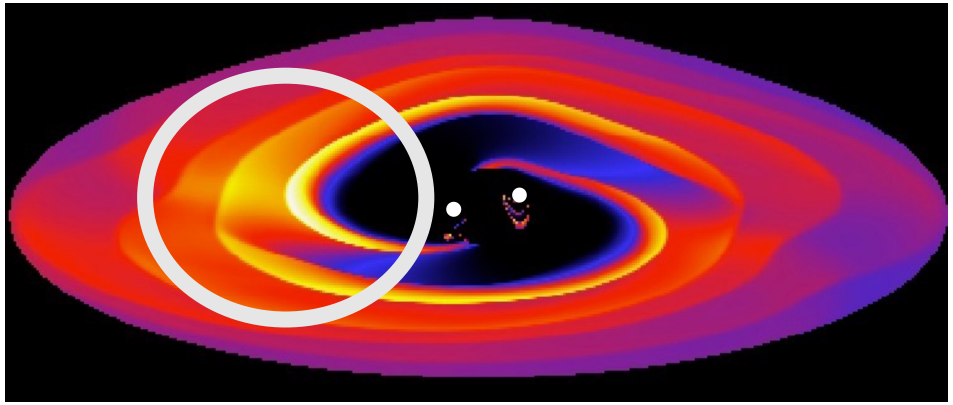
Tiede+20, 2D Hydro

Synthetic observations of pre-merger BBHs

- **GR ray-tracing** code (Vincent+11) incorporating the **BBH approximate metric** (Ireland+16)
- Thermal emission, thin disk approximation (Shakura & Sunyaev, 1973)
- Putting physical units back: mass scaling from Lin+13 ($M = 10^5 M_\odot$; $T_{in} = 0.1$ keV) as reference

➢ Obtain the multi-wavelength emission map
  - ➢ The metric evolves as photons propagate
  - ➢ Emission map composed of photons of different time-origin (hence, fluid outputs!)



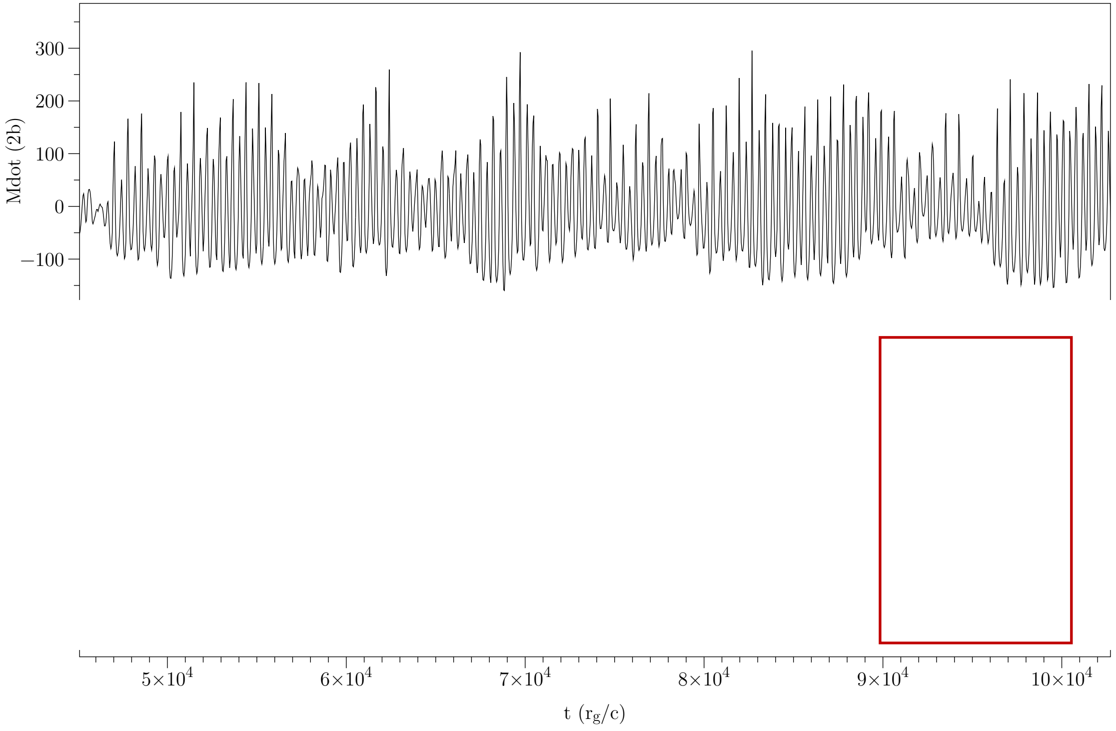


Impact of the lump & spiral arms

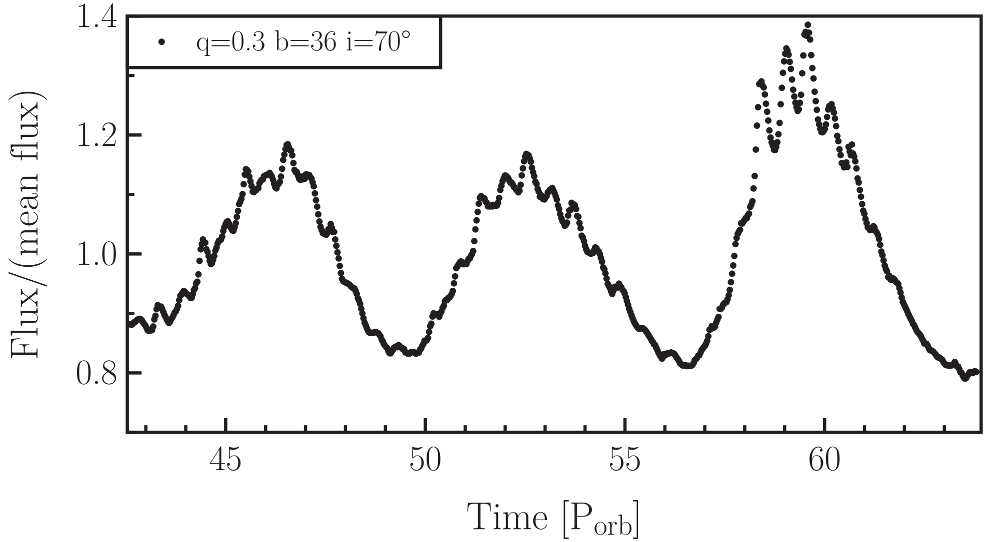
Timing features

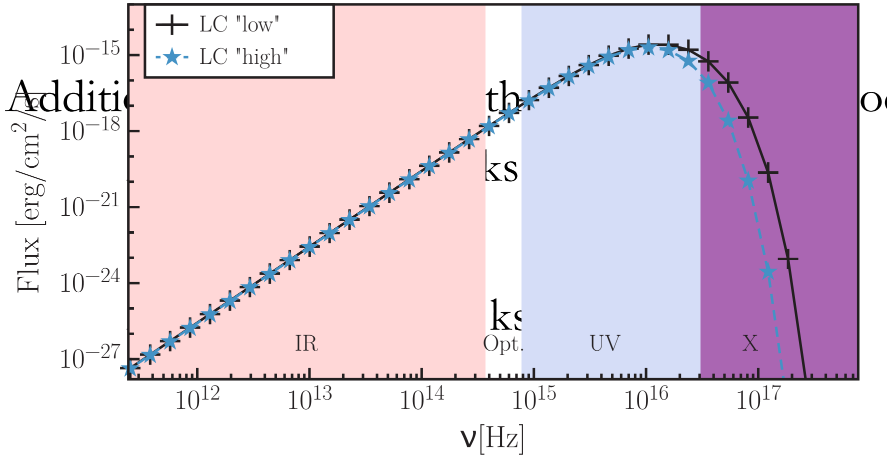
- Accretion rate: proxy for the luminosity? (e.g. Krauth+23)

$q = 0.1;\ b = 20r_g$



$q = 0.3;\ b = 36r_g$



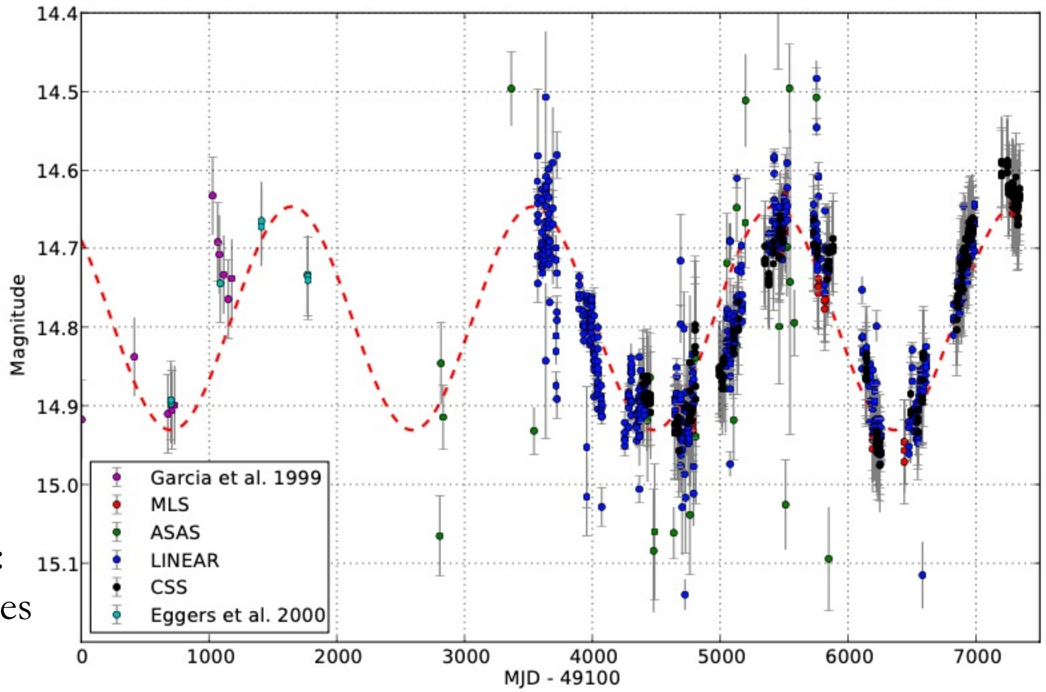


- Additi th od ks ks

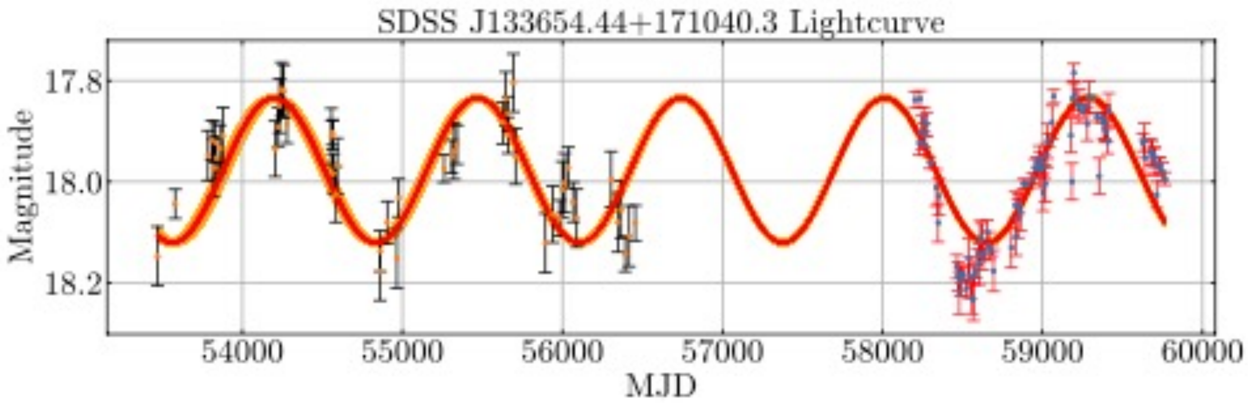
Double EM variability: the signature of circumbinary disks around BBHs? (MR+subm.)

The hunt for binary black holes is now open !

Graham+15:
111 candidates



- LISA will be launched in ~10 years...
- Let us confront the predictions from numerical simulations with actual observational data
- Are these consistent with a binary black hole ?



So far, still no unambiguous detection of a binary black hole in electromagnetic light

Foustoul, Webb, MR et al., submitted: advanced selection in the Graham+15 catalog

Conclusions: observational features of BBH circumbinary disks

- GR-hydrodynamics is the main tool to study the environment of single/binary black holes

- New results on binary black holes:

  1. Accretion structures typical of BBHs: streams+spiral arms, cavity, «lump» (e.g. Noble+12, Shi+12)
  (MR+23, MNRAS)
  2. Accretion rate variability at twice the orbital-lump beat frequency

  3. Observational consequences in thermal emission:

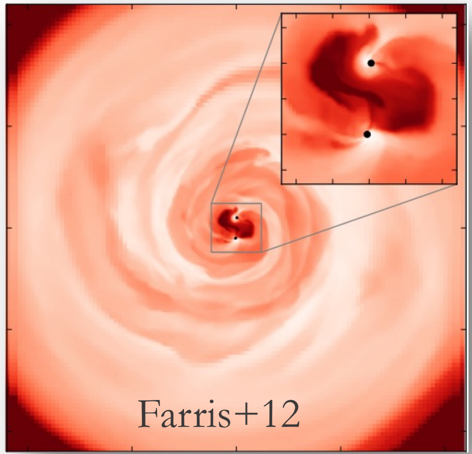
  ➢ Double variability in the lightcurve, dominated by the «lump» modulation

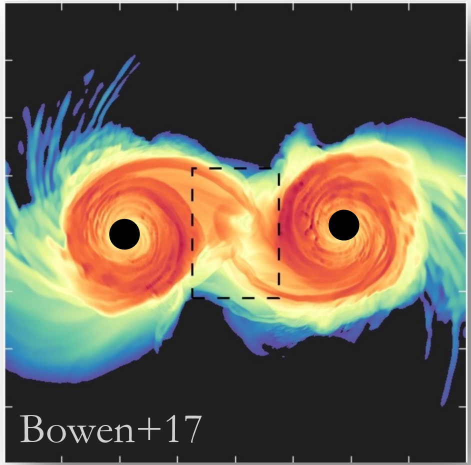
    ➢ Accretion rate is <u>not</u> a good proxy for the luminosity: **GR ray-tracing is mandatory**
    (MR+subm. to MNRAS)

- The unambiguous detection of a BBH in electromagnetic light is still lacking... GR-hydro is needed

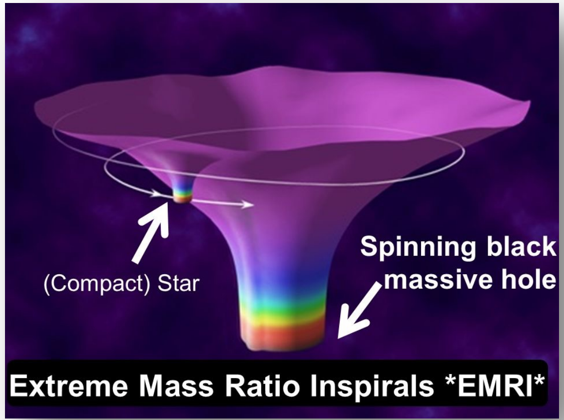
Perspectives and possible projects

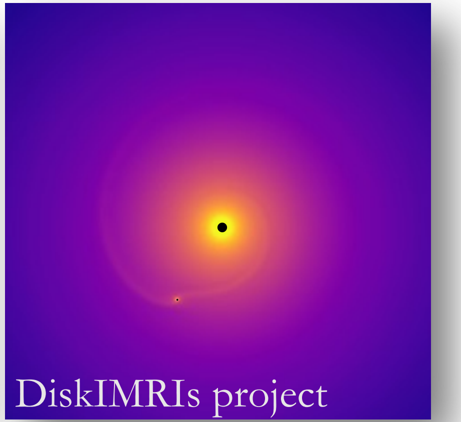
- Accretion/ejection structures in binary black holes



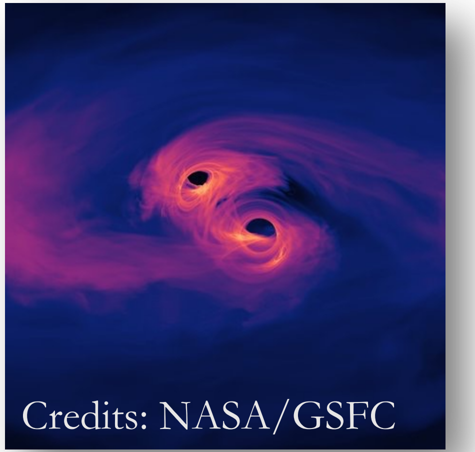


- « Extreme-mass-ratio inspiral » (EMRIs) in disks



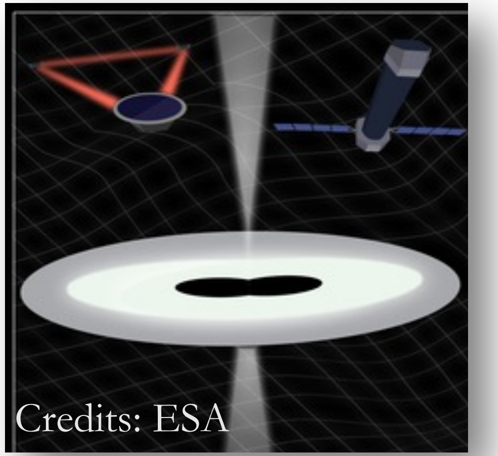


- Prediction of electromagnetic signatures



This is NOT an artiste view

- Optimization of future GW-electromagnetic observations



- Acceleration of high-energy particles & neutrinos production around BBHs