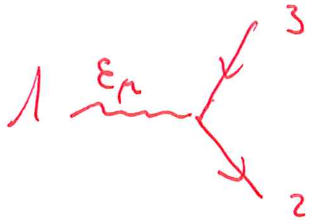
Z - decay



$$g_Z = \frac{g}{\cos\theta_W}$$

$$i\mathcal{M} = \varepsilon_\mu \, \bar{u}(2) \left[-\frac{ig}{2\cos\theta_W} \gamma^\mu (g_V - g_A \gamma^5) \right] v(3)$$

$$|\mathcal{M}|^2 = \left(\frac{g_Z}{2}\right)^2 \varepsilon_\mu \varepsilon_\nu^* \left[\bar{u}(2) \gamma^\mu (g_V - g_A\gamma^5) v(3) \right]$$
$$\left[\bar{v}(3) \gamma^0 \left(\gamma^\mu (g_V - g_A \gamma^5) \right)^\dagger \gamma^0 u(2) \right]$$

$$\gamma^0 \left(\gamma^\mu (g_V - g_A\gamma^5)\right)^\dagger \gamma^0 = \gamma^0 (g_V - g_A \gamma^5)^\dagger \gamma^{\nu\dagger} \gamma^0$$

$$= (g_V + g_A \gamma^5) \gamma^0 \gamma^{\nu\dagger} \gamma^0 \qquad \gamma^{5\dagger} = \gamma^5 \qquad \gamma^0 \gamma^{\nu\dagger} \gamma^0 = \gamma^\nu$$

$$= (g_V + g_A\gamma^5)\gamma^\nu = \gamma^\nu (g_V - g_A \gamma^5)$$

$$|\mathcal{M}|^2 = \left(\frac{g_Z}{2}\right)^2 \varepsilon_\mu \varepsilon_\nu^* \left[\bar{u}(2) \gamma^\mu (g_V - g_A\gamma^5) v(3) \right]$$
$$\left[\bar{v}(3) \gamma^\nu (g_V - g_A\gamma^5) u(2) \right]$$

the same V-A structure

$$\sum_{s_1,s_2} [\ \ ][\ \ ] = \mathrm{tr}\left\{\gamma^\mu (g_V - g_A\gamma^5)(\not{p}_3 + m)\gamma^\nu (g_V - g_A\gamma^5) \times (\not{p}_2 + m_2)\right\}$$

$$= 4(g_V^2 + g_A^2)\left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - (p_2 \cdot p_3)\eta^{\mu\nu}\right]$$

$$+ 4(g_V^2 - g_A^2)\, m^2 \eta^{\mu\nu} - 8i\, g_V g_A\, \varepsilon^{\mu\nu\lambda\sigma} p_{2\lambda} p_{3\sigma}$$

$$\frac{1}{3}\sum_r \varepsilon_\mu \varepsilon_\nu^* = \left(-\eta_{\mu\nu} + \frac{p_{1,\mu} p_{1,\nu}}{m_Z^2}\right)/3$$

$\varepsilon^{\mu\nu\lambda\sigma} = -\varepsilon^{\nu\mu\lambda\sigma}$ but $p_{1,\mu} p_{1,\nu}$ & $\eta_{\mu\nu}$ symmetr.

$$|\bar{\mathcal{M}}|^2 = \frac{1}{3} g_Z^2 \left(-\eta_{\mu\nu} + \frac{p_{1\mu} p_{1\nu}}{m_Z^2}\right) \overbrace{\ \ }^{(g_V^2+g_A^2)} \left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - (p_2 \cdot p_3)\eta^{\mu\nu}\right]$$

$$+ \mathcal{O}(m^2/m_Z^2) \sim 10^{-3} \text{ for } m_b$$

$$= \frac{1}{3} g_Z^2 (g_V^2 + g_A^2)\Big\{-2(p_2 \cdot p_3) + 4(p_2 \cdot p_3)$$

$$+ \frac{1}{m_Z^2}\Big[2(p_1 \cdot p_2)(p_1 \cdot p_3) - \underset{\overset{\parallel}{m_Z^2}}{p_1^2}(p_2 \cdot p_3)\Big]\Big\}$$

$$= \frac{1}{3} g_Z^2 (g_V^2 + g_A^2) \left\{ (p_2 \cdot p_3) + \frac{2(p_1 \cdot p_2)(p_1 \cdot p_3)}{m_Z^2} \right\}$$

decay width $\quad d\Gamma = \frac{1}{2E_i} |\mathcal{M}_{fi}|^2 \, d\Phi^{(n)}$

$$d\Phi^{(2)} = (2\pi)^4 \, \delta(M - E_2 - E_3) \, \delta^{(3)}(\vec{p}_2 + \vec{p}_3) \, \frac{d^3p_2}{2E_2 (2\pi)^3} \, \frac{d^3 p_3}{2E_3 (2\pi)^3}$$

$$= \frac{1}{(2\pi)^2} \frac{1}{4E_1E_2} \, \delta(M - E_1 - E_2) \, d^3p_3$$

$$d^3p_3 = d\Omega \, p_3^2 \, dp_3$$

$$M - \sqrt{m_2^2 + p_3^2} - \sqrt{m_3^2 + p_3^2} \equiv M - x$$

$$\frac{dx}{dp_3} = \frac{p_3 \, x}{E_1 E_2} \qquad \Leftarrow \frac{p_3}{E_2} + \frac{p_3}{E_3} = \frac{p_3(E_2 + E_3)}{E_2 E_3}$$

$$= \frac{1}{16\pi^2} \, d\Omega \int_0^\infty dx \, \frac{p_3}{x} \, \delta(M - x) = \frac{1}{16\pi^2} \, d\Omega \, \frac{|\vec{p}_3|}{M}$$

$$d\Omega \to 4\pi$$

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{|\vec{p}_3|}{M^2} |\overline{\mathcal{M}}|^2$$

$|\overline{\mathcal{M}}|^2$: isotropic $\qquad \Gamma = \frac{1}{8\pi} \frac{|\vec{p}|}{M^2} |\overline{\mathcal{M}}|^2$

rest frame of Z:

$$p_1 = (m_Z, \vec{0}) \,, \quad p_2 = (m_Z/2, \vec{p}) \,, \quad p_3 = (m_Z/2, -\vec{p})$$

$$p_2^2 = p_3^2 = 0 \quad \text{for } m_f \to 0$$

$$|\vec{p}| = m_Z/2$$

$$p_2 \cdot p_3 = \left(\frac{m_Z}{2}\right)^2 + \left(\frac{m_Z}{2}\right)^2 = \frac{m_Z^2}{2} = p_1 \cdot p_3 = p_1 \cdot p_2$$

$\Rightarrow$

$$\{ \quad \} = \frac{m_Z^2}{2} + \frac{2 \, m_Z^2/2 \; m_Z^2/2}{m_Z^2} = m_Z^2$$

$$\Gamma = \frac{1}{8\pi} \frac{m_Z/2}{m_Z^2} \frac{1}{3} g_Z^2 (g_V^2 + g_A^2) \cdot m_Z^2$$

$$= \frac{m_Z}{48\pi} \left(\frac{g}{\cos\vartheta_0}\right)^2 (g_V^2 + g_A^2)$$

set $\varepsilon = g_A / g_V$, $\mathrm{tr}(\# \gamma) = 0$:

$$\mathrm{tr}\{\ldots\} = g_V^2\, \mathrm{tr}\left\{ \gamma^\mu (1-\varepsilon\gamma^5) \not{p}_3\, \gamma^\nu (1-\varepsilon\gamma^5) \not{p}_2 \right\}$$

$$- m_1 m_3\, g_V^2\, \mathrm{tr}\left\{ \gamma^\mu (1-\varepsilon\gamma^5)\, \gamma^\nu (1-\varepsilon\gamma^5) \right\} = g_V^2 (T_1 - m_1 m_2 T_2)$$

$$T_2 = \mathrm{tr}(\gamma^\mu \gamma^\nu) + \varepsilon^2\, \mathrm{tr}(\gamma^\mu \gamma^5 \gamma^\nu \gamma^5) \qquad : \mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^5) = 0$$

$$= 4\eta^{\mu\nu} (1-\varepsilon^2)$$

$$T_1 = \mathrm{tr}(\gamma^\mu \not{p}_3 \gamma^\nu \not{p}_2) - \varepsilon\, \mathrm{tr}(\gamma^\mu \gamma^5 \not{p}_3 \gamma^\nu \not{p}_2)$$

$$- \varepsilon\, \mathrm{tr}(\gamma^\mu \underbrace{\not{p}_3}_{+} \underbrace{\gamma^\nu}\, \gamma^5 \not{p}_2)$$

$$+ \varepsilon^2\, \mathrm{tr}(\gamma^\mu \gamma^5 \not{p}_3 \gamma^\nu \gamma^5 \not{p}_2)$$

$$\to + \varepsilon^2\, \mathrm{tr}(\gamma^\mu \not{p}_3 \gamma^\nu \not{p}_2)$$

$$\mathrm{tr}(\gamma^\mu \not{p}_3 \gamma^\nu \not{p}_2) = 4 (p_3^\mu p_2^\nu - \eta^{\mu\nu} p_2 \cdot p_3 + p_2^\mu p_3^\nu)$$

$$\mathrm{tr}(\gamma^5 \not{p}_3 \gamma^\nu \not{p}_2 \gamma^\mu) = p_{3,\alpha}\, p_{2,\beta}\, \underbrace{\mathrm{tr}(\gamma^5 \gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu)}_{4i\, \varepsilon^{\alpha\nu\beta\mu}}$$

$$= 4i\, \varepsilon^{\alpha\nu\beta\mu}\, p_{3,\alpha}\, p_{2,\beta}$$

$$T_1 = 4(1+\varepsilon^2)\left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - \eta^{\mu\nu}\, p_2 \cdot p_3\right]$$

$$- 8 i \varepsilon^{\alpha\nu\beta\mu} \, p_{3,\alpha}\, p_{2\beta}$$

$\Rightarrow$

$$\mathrm{tr}\{\dots\} = 4(g_V^2+g_A^2)\left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - \eta^{\mu\nu}\, p_2 \cdot p_3\right]$$

$$+ 4(g_V^2 - g_A^2)\, m_2 m_3\, \eta^{\mu\nu} - 8 i \varepsilon^{\alpha\nu\beta\mu}\, p_{3,\alpha}\, p_{2,\beta}$$

Z-resonance

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma + \mathcal{M}_Z| \overset{s\approx m_Z^2}{\approx} |\mathcal{M}_Z|^2$$

with $\mathcal{M}_Z = \dfrac{g_Z^2}{s-m_Z^2+im_Z\Gamma_Z} \dfrac{1}{4}\left[\bar{v}(p_2)\gamma^\mu(g_V^e - g_A^e\gamma^5)\,u(p_1)\right]$
$\left[\quad 4 \quad (g_V^f - g_A^f\gamma^5) \quad 2 \quad\right]$

$$\sigma(e^+e^- \to Z \to \bar{f}f) = \frac{1}{192\pi}\,\frac{g_Z^4\, s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}\left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^f)^2 + (g_A^f)^2\right]$$

$$\Gamma(Z \to \bar{f}f) = \frac{g_Z^2 m_Z^2}{48\pi}\left[(g_V^f)^2 + (g_A^f)^2\right]$$

$$\Rightarrow \quad \sigma = \underbrace{\frac{12\pi s}{m_Z^2}}_{C}\,\frac{\Gamma(Z\to e^+e^-)\,\Gamma(Z\to\bar{f}f)}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

this holds in general with

$$C = \frac{4\pi s}{p_{cm}^2}\,\frac{(2s_R+1)}{(2s_1+1)(2s_2+2)} \qquad \text{d}Z = \frac{4\pi s \cdot 4}{s}\,\frac{3}{4} = 12\pi$$

Γ is also half-width