Exercise sheet 3

1. Measurements.
The universe is filled with cosmic microwave background (CMB) photons. Assume for simplicity that they have a single energy ω_0. Which energy measures the uniformly accelerated observer from the last exercise sheet at time τ?

2. Line-element.
Show that the line-element

$$\mathrm{d}s^2 = \mathrm{d}t^2 - 2\mathrm{d}x\,\mathrm{d}t - \mathrm{d}y^2 - \mathrm{d}z^2$$

corresponds to a flat spacetime.

3. Cylinder coordinates I.
Calculate for cylinder coordinates $x = (\rho, \phi, z)$ in $\mathbb{R}^3$

$$\begin{aligned} x_1' &= \rho\cos\phi\,, \\ x_2' &= \rho\sin\phi\,, \\ x_3' &= z\,, \end{aligned}$$

the basis vectors $\boldsymbol{e}_i$, the components of g_{ij} and g^{ij}, and $g \equiv \det(g_{ij})$.

4. Hyperbolic plane H^2
The line-element of the hyperbolic plane H^2 is given by

$$\mathrm{d}s^2 = y^{-2}(\mathrm{d}x^2 + \mathrm{d}y^2) \qquad \text{and} \quad y \geq 0\,.$$

a.) Show that points on the x-axis are an infinite distance from any point (x, y) in the upper plane. [The length s of a line between a and b along x is given by $s = \int_a^b \sqrt{g_{xx}}$.]
b.) Deduce the Christoffel symbols $\Gamma^a{}_{bc}$.
c.) Write out the geodesic equations and solve them to find x and y as function of the length s of these curves.

5. Killing vector fields of Minkowski space
Find the Killing vector fields of Minkowski space and specify the corresponding symmetries and conserved quantities. [Hint: Differentiate the Killing equation, permute the indices and find an equation for a single term which you can integrate.]

Solutions are discussed Thursday, 01.02.24