Exercise sheet 4

1. Light deflection?
In another theory of gravity, the metric outside a star is

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\left[\mathrm{d}t^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\vartheta^2 + \sin^2\vartheta \mathrm{d}\phi^2)\right] .$$

a.) Calculate the deflection of photons in this theory.
b.) Use a general argument why you should get this result.

2. Circular orbits
A spaceship is moving without power in a circular orbit about a star with mass M. The radius in Schwarzschild coordinates is $r = 7M$.
a.) Show that $\Omega = \mathrm{d}\phi/\mathrm{d}t = (M/r^3)^{1/2}$.
b.) What is the period measured by an observer at infinity?
c.) What is the period measured by a clock onboard the spaceship?

3. Falling into a BH
An observer falls radially into a Schwarzschild BH of mass M. The observer starts from rest relative to a stationary observer at $r = 10M$. How much time elapses on the clock of the falling observer before hitting the singularity at $r = 0$?

4. Force of a hovering rocket.
A stationary observer hovers on a radial orbit around a Schwarzschild black-hole.
a.) Argue that

$$f^\alpha = m\left(\ddot{x}^\alpha + \Gamma^\alpha{}_{\beta\gamma}\dot{x}^\beta\dot{x}^\gamma\right)$$

is the correct generalisation of Newton's second law to curved spacetime.
b.) Using $\Gamma^r{}_{tt} = (1 - 2M/r)(M/r^2)$, find the radial force required to stay on a stationary orbit.
c.) The result from b.) is the radial force in the coordinate basis. Relate it to the force measured by the observer in its Cartesian inertial frame.

Solutions are discussed Thursday, 15.02.24