

Exercise sheet 5**1. Perturbation of unstable circular orbit**

A small perturbation of an unstable circular orbit around a Schwarzschild BH will grow (initially) exponentially with time. Show that a displacement δr will grow as $\delta r \propto \exp(\tau/\tau_*)$, where τ is the proper-time along the trajectory. Evaluate τ_* and explain its behavior for $r_{\text{max}} \rightarrow 6M$.

2. Survival time.

What is the maximal proper-time an observer can enjoy inside a Schwarzschild black-hole after crossing the horizon?

3. Entropy and evaporation time of BHs

- a.) Calculate the (photon) luminosity and the life-time of a Schwarzschild BH, if its temperature is $T = 1/(8\pi M)$. Assume that the area from which the photons are emitted is the horizon area. What is the mass of a primordial (i.e. created at time $t \simeq 0$) BH that evaporates today?
- b.) What are potential detection methods?
- c.) Estimate the entropy of a $10M_{\odot}$ star before collapse and compare it to the one of the resulting BH.
- d.) Try to estimate the total entropy of the observable universe.