Exercise sheet 6

1. Scalar electrodynamics.
a.) Write down the Lagrangian of scalar QED, i.e. a complex scalar field coupled to the photon via $D_\mu = \partial_\mu + \mathrm{i}qA_\mu$. Derive the Noether current(s) and the current to which the photon couples (defined by $\partial_\mu F^{\mu\nu} = j^\nu$).
b.) Show that $\mathscr{L} = -F^2/4$ corresponds to a canonically normalised field, i.e. that

$$\mathscr{L} = -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\, A^\mu D_{\mu\nu} A^\nu,$$

where $D_{\mu\nu}$ is a differential operator.

2. Dimension of a scalar field ϕ.
We set $\hbar = c = 1$ and choose as the unique dimension the mass unit m.
a.) Find the dimension (i.e. the power m^α) of a scalar field ϕ in d spacetime dimensions.
b.) For which d has $\mathscr{L}_I = \lambda\phi^3$ a dimensionless coupling constant?
c.) For which d has $\mathscr{L}_I = \lambda\phi^4$ a dimensionless coupling constant?

3. Stress tensor for the electromagnetic field.
a.) Determine the stress tensor $T^{\mu\nu}$ of the electromagnetic field using either i) $f^\mu = -\frac{\partial T^{\mu\nu}}{\partial x^\nu}$, where $T^{\mu\nu}$ is the stress tensor of the field acting with the force density f_μ on external currents, or ii) converting $T^{00} = (\boldsymbol{E}^2 + \boldsymbol{B}^2)/2$ into an valid tensor expression.
b.) Confirm that T^{00} corresponds to the energy-density ρ. Find the trace of $T^{\mu\nu}$ and the Equation of State (EoS) defined by $w = P/\rho$.

Solutions are discussed Thursday, 29.02.24