Exercise sheet 6

1. “Adjoint” gamma matrices $\tilde{\Gamma}$
   a.) Show that $\gamma^0 \gamma^\mu \gamma^0 = \gamma^\mu$.
   b.) Show for any product of $\gamma^\mu$ matrices, $\Gamma = \gamma_1 \cdots \gamma_n$ that $\tilde{\Gamma} \equiv \gamma^0 \Gamma \gamma^0$ is the reverse product, $\tilde{\Gamma} = \gamma_n \cdots \gamma_1$.
   c.) Calculate $\tilde{\Gamma}$ for $\Gamma = \gamma^\mu (1 - \gamma^5)$.

2. Compton scattering
   Write down the Feynman amplitude $\mathcal{M}$ for Compton scattering $e^-\gamma \rightarrow e^-\gamma$. Show that the two diagrams are connected by the crossing relation $\varepsilon_1 \leftrightarrow \varepsilon_2^*$ and $k_1 \leftrightarrow -k_2$.

3. Physical photon states.
   a.) Let $A_\mu (x) = \sum_r a_r \varepsilon^{(r)}_\mu e^{-ikx}$ be a solution of the wave-equation for the photon, where $a_r$ denotes the amplitude of the polarisation state $r$. Find the possible wave-vectors $k_\mu$ and polarisation vectors $\varepsilon^{(r)}_\mu$ of a photon with energy $\omega$ moving in the $z$ direction. How do they transform under Lorentz transformations? [Hint: Consider gauge transformation

   \[ A_\mu (x) \rightarrow A'_\mu (x) = A_\mu (x) + \partial_\mu \Lambda (x) \] (1)

   of the type $\Lambda (x) \propto \lambda \exp (-ikx)$ and use that the physical photon states cannot depend on the arbitrary parameter $\lambda$.]
   b.) From the result in a.), you can conclude that any amplitude $\mathcal{M} = \varepsilon_\mu \mathcal{M}^\mu$ vanishes, if the polarisation vector $\varepsilon_\mu$ of a photon is replaced by its momentum vector, $k_\mu \mathcal{M}^\mu = 0$.

4. Form factor of the proton
   Show that using $q_\mu K^{\mu\nu} = 0$ in

   \[ K^{\mu\nu}(p, q) = A p^{\mu} q^{\nu} + B p^{\nu} q^{\mu} + C q^{\mu} q^{\nu} + D (p^{\mu} q^{\nu} + q^{\mu} p^{\nu}) \]

   allows us to express $K^{\mu\nu}$ by only two functions, say $A$ and $B$.

Solutions are discussed Thursday, 06.10.16