Exercise sheet 8

0. Green function of the wave equation.
Recall from your electrodynamics lectures the derivation of the Green function of the wave equation.

1. Hyperbolic plane H^2.
Calculate the Riemann (or curvature) tensor $R^a{}_{bcd}$ and the scalar curvature R for the hyperbolic plane H^2.

2. Spin and helicity of tensor fields.
A plane wave ψ which is transformed into $\psi' = \mathrm{e}^{-\mathrm{i}h\alpha}\psi$ by a rotation with angle α around its propagation axis is said to have helicity h. Specifically, consider a photon and a graviton propagating into the z direction, $\boldsymbol{k} = k\boldsymbol{e}_z$, and choose the rotation in the xy plane. Start with linear polarised states,

$$A^\mu = \varepsilon^\mu \mathrm{e}^{-\mathrm{i}kx}, \qquad h^{\mu\nu} = \varepsilon^{\mu\nu}\mathrm{e}^{-\mathrm{i}kx},$$

where the two polarisation vectors are $\varepsilon^{(1)}_\mu = \delta^1_\mu$ and $\varepsilon^{(2)}_\mu = \delta^2_\mu$ for the photon, and the polarisation tensors for the graviton are

$$\varepsilon^{\mu\nu}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \varepsilon^{\mu\nu}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Rotate the states, introduce circular polarised states and find their helicity. [Remark: Helicity is the projection of spin on the momentum: Thus this exercises shows that the photon (or generally a vector field) is a spin-1, while the graviton is a spin-2 particle.]

Solutions are discussed Thursday, 21.03.24