Electroweak corrections to DM annihilations

Michael Kachelrieß

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Introduction

- Motivation
- Recap: LLO and MLLO corrections in QCD
- 2 Electroweak bremsstrahlung
- Operation of the second sec
 - Hidden annihilations $XX \rightarrow \nu\nu$
 - Limits on leptophilic models $XX \rightarrow e^+e^-$

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DM annihilations and PAMELA

DM with M = 1 TeV that annihilates into $\mu^+ \mu^-$



non-standard branching ratios: only leptons

- boost factor 1000 needed
- but minimal γ -ray flux from Bremsstrahlung not seen

DM annihilations and PAMELA

DM with M = 10 TeV that annihilates into W^+W^-



standard branching ratios:

- hide \bar{p} above E_{\max} of Pamela
- happy with M = 10 TeV?

WIMPs as thermal relics:

• expansion of universe freezes out annihilation reactions, when

 $\Gamma_{\rm ann} = n \langle \sigma_{\rm ann} v \rangle \approx H$

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⇒ "WIMP miracle:" suggests weakly interacting DM particle with mass $m \sim m_Z$

• thermally averaged annihilation cross section

$$\langle \sigma_{\mathrm{ann}} v \rangle = \sigma_0 + \sigma_1 v^2 + \sigma_2 v^4 + \dots$$

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- known value of Ω_X fixes σ_0 today

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• Multi-TeV DM is interacting pretty strongly

• annihilation $XX \rightarrow \bar{f}f$ via t exchange of SU(2) scalar



• annihilation $XX \rightarrow \overline{f}f$ via t exchange of SU(2) scalar for Majorana DM:



• annihilation $XX \rightarrow \bar{f}f$ via t exchange of SU(2) scalar



• for $v_{\rm rel}
ightarrow 0$, neutralino pair behaves as a pseudoscalar,

$$u_1(P)\bar{v}_2(P) - u_2(P)\bar{v}_1(P) = (m_{\chi} + \not\!\!\!P)\gamma_5 \quad , \quad | \to \rangle | \leftarrow \rangle , \quad -| \leftarrow \rangle | \to \rangle,$$
$$u_1(P)\bar{v}_2(P) - u_2(P)\bar{v}_1(P) = \mathbf{0} \quad , \quad | \to \rangle | \to \rangle, \quad | \leftarrow \rangle | \leftarrow \rangle.$$

 $\bullet\,$ for simplicity, $y\equiv M_S^2/M_X^2\ll 1$, then

$$\mathcal{L}_{\text{eff}} = \mathcal{O}_4 + \frac{y^2}{M_X^2}\mathcal{O}_6 + \frac{y^4}{M_X^4}\mathcal{O}_8 + \dots$$

with

$$\mathcal{O}_6 = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{f}\gamma^\mu\gamma_5f)\,.$$

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$$\bar{\chi}\gamma_{\mu}\gamma_{5}\chi \to \frac{P_{\mu}}{\sqrt{2}m_{X}}\bar{\chi}\gamma_{5}\chi$$

• \mathcal{O}_6 becomes divergence of axial-vector current,

$$\mathcal{O}_6 \to \left[\frac{i}{\sqrt{2}M_X}\bar{\chi}\gamma_5\chi\right] \left[\partial_\mu(\bar{f}\gamma^\mu\gamma_5 f)\right] \propto \frac{m_f}{M_X}.$$
 (1)

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Neutralino annihilations

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- \Rightarrow annihilations into b,t quarks and $W, ec{Z}, h, H, A$
- typical hadronization spectra with $\varphi_{\nu}(E)/2 \sim \varphi_{\gamma}(E) \sim 3\varphi_e(E) \sim 10\varphi_N(E)$





• denominator of the additional fermion propagator is

$$\frac{A(k,p)}{(p-k)^2 - m^2} = \frac{A(k,p)}{-2p \cdot k} \approx \frac{A(k,p)}{-2E\omega(1-\cos\vartheta)}$$

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- blows up for emission of
 - soft gluons, $\omega \to 0$.
 - collinear emission of gluons, $\vartheta \to 0$.
- \Rightarrow compensate the small coupling $lpha_s(Q^2)/\pi \ll 1$

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Sudakov form factor

• interference terms disappear in "classical limit"

Sudakov form factor

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- factorisation, exponentiate log's

$$|\mathcal{M}(s,t)|^2 = |\mathcal{M}_{\text{Born}}(s,t)|^2 \exp\left(-\int \frac{d\tilde{t}}{2\pi\tilde{t}} \alpha(\cdot) \int dz P_{ij}(z)\right)$$

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Sudakov form factor

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- Using only FSR,

$$R = \frac{\Gamma(X \to \bar{\nu}\nu Z)}{\Gamma(X \to \bar{\nu}\nu)} = \frac{\alpha_2}{8\pi c_W^2} \left(\ln^2 \epsilon + 3\ln \epsilon + \ldots\right),$$
 where $\varepsilon = \left(\frac{m_Z}{M_X}\right)^2$

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- $M_X \gtrsim 10^6$ GeV, \Rightarrow naive perturbation theory breaks down: electroweak and SUSY sector have a QCD-like behavior ("jets")
- (modified) DGLAP description possible

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DESY, 17. June 2011

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Fragmentation of heavy particles $X \rightarrow \nu \nu$



for $m_X=10^{16},\,10^{13}$ and $10^{10}~{\rm GeV}$

Electroweak bremsstrahlung: Sudakov form factors

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mass corrections change only subleading log's

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- mass corrections change only subleading log's
- but $\sigma_{\rm NLO}/\sigma_{\rm LO}\sim 50\%$ at LHC: something missed?

TeV DM and electroweak bremsstrahlung

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Perturbative unitarity and multi-TeV masses

- 2 well-known extremes: SM and MSSM
 - SM: masses \gtrsim TeV break unitarity, e.g. $Z_L ar{f} f \propto m_f/m_Z$

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- 2 well-known extremes: SM and MSSM
 - SM: masses \gtrsim TeV break unitarity, e.g. $Z_L \bar{f} f \propto m_f/m_Z$
 - MSSM: decouples for $M_{\rm SUSY}
 ightarrow \infty$, e.g. $Z_L \chi_1 \chi_1 \propto m_Z/m_\chi$

$$O_{11}^Z \frac{m_{\chi}}{m_Z} = -\frac{1}{2} O_{11}^G \, ,$$

in terms of the neutralino mixings and masses,

$$\begin{aligned} & (N_{14}^2 - N_{13}^2) \frac{m_{\chi}}{m_Z} = \\ & = -(c_W N_{12} - s_W N_{11}) (s_\beta N_{14} + c_\beta N_{13}) \end{aligned}$$

• choose a SUSY inspired simple model

Toy model for electroweak bremsstrahlung

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- DM χ is singlet majorana, SU(2) scalar η
- $\chi L_e \eta$ and $\Phi \eta$ interactions,

$$\mathcal{L}_{\text{int}}^{\text{fermion}} = f\bar{\chi}(L_e i\sigma_2 \eta) + \text{h.c.} = f\bar{\chi}(\nu_{eL}\eta^0 - e_L\eta^+) + \text{h.c.} ,$$
$$\mathcal{L}_{\text{int}}^{\text{scalar}} = -\lambda_3(\Phi^{\dagger}\Phi)(\eta^{\dagger}\eta) - \lambda_4(\Phi^{\dagger}\eta)(\eta^{\dagger}\Phi) .$$

Theoretical problems

Toy model for electroweak bremsstrahlung

- DM χ is singlet majorana, SU(2) scalar η
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• effective operator approach

$$\mathcal{L}_{\text{eff}} = \mathcal{O}_4 + \frac{y^2}{M_X^2}\mathcal{O}_6 + \frac{y^4}{M_X^4}\mathcal{O}_8 + \dots$$

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• leading behavior of the amplitude

$$\mathcal{M} \sim \frac{\mathcal{O}(v)}{M_{\chi}} \left[\mathcal{O}(y) |_{\text{FSR}} + \mathcal{O}(y^2) |_{\text{FSR}} \right] + \left[\mathcal{O}(y^2) |_{\text{VIB}} + \mathcal{O}(y^2) |_{\text{FSR}} \right]$$

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 \Rightarrow s-wave appears only at $\mathcal{O}(y^2)$

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- $\Rightarrow\,$ s-wave appears only at $\mathcal{O}(y^2)$
- \Rightarrow cross section

$$v\sigma(\chi\chi \to f\bar{f}Z) \sim \frac{\alpha_W}{M_\chi^2} \left[\mathcal{O}\left(v^2y^2\right) + \mathcal{O}\left(v^2y^3\right) + \mathcal{O}\left(y^4\right) \right] \,,$$

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Hidden annihilations $XX \rightarrow \nu\nu$

How does electroweak Bremsstrahlung affect the limit?

- peak $\delta(E-m_X)$ becomes smoother
- contribution to diffuse $\gamma\text{-}\mathrm{ray}$ background

How to handle model-dependence?

- for bound: use only FSR in a model where W_L does not contribute







Toy model $XX \rightarrow e^+e^-$

[Bell et al. '11]



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Importance of VIB as function of y:





[Garny, Ibarra, Vogl '11]



[Garny, Ibarra, Vogl '11]



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[Garny, Ibarra, Vogl '11]



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 - \bar{p} and γ fluxes can give stringent limits on leptophilic models
 - electromagnetic bremsstrahlung features offer DM signature
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