Exercise sheet 3

Hartle 7-14.
Newton's law becomes in relativistic notation

$$\mathbf{F} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}\tau}\,.$$

Calculated $\mathbf{F}\cdot\mathbf{u}$ taking into account that the mass m changes. Why do we set in general $m = \text{const.}$?

Hartle 7-14.
Consider the metric

$$ds^2 = -(1-Ar^2)^2dt^2 + (1-Ar^2)^2dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta\mathrm{d}\phi^2)$$

i) Calculate the proper distance from $r = 0$ to $r = R$.
ii) Calculate the area of a sphere with coordinate radius R.
iii) Calculate the three-volume of a sphere with coordinate radius R.
iv) Calculate the four-volume of a sphere with coordinate radius R bounded by two t=const planes separated by time difference T.

Spherical coordinates II.
Calculate for spherical coordinates $x = (r, \vartheta, \phi)$ in $\mathbb{R}^3$ the gradient, divergence, and the Laplace operator. Note that one uses usually normalized unit vectors in case of a diagonal metric: this corresponds to a rescaling of vector components $V^i \to V^i/\sqrt{g_{ii}}$ (no summation in i) or basis vectors.

Solutions are discussed Monday, 08.02.10