Exercise sheet 4

Hartle 7-9.
In a (pseudo-) Riemannian manifold, one can find in each point P a coordinate system, called (Riemannian) normal or geodesic coordinates, with the following properties,

$$g'_{ab}(P) = \eta_{ab} \tag{1}$$
$$\partial_{c'} g'_{ab}(P) = 0 \tag{2}$$
$$\Gamma^a_{bc}(P) = 0 \tag{3}$$

Expand the coordinates $x^\alpha(\bar{x}^\beta)$ around the point x_P up to third order. Show that there is enough freedom in coordinate transformations to make the first derivative vanish but not the second.

Fermat's principle of Least Time - Hartle 8-14.
Consider a medium with index of refraction $n(\vec{x})$, i.e. the speed of light varies as $c/n(\vec{x})$. Fermat's principle states that light rays follows paths between two space points that minimize the travel time.
a.) Show that the paths of least time are geodesics in space with line-element

$$ds^2 = n^2(dx^2 + dy^2 + dz^2)$$

b.) Write out the geodesic equation in (x, y, z) coordinates.

Hartle 9.8
A spaceship is moving without power in a circular orbit about a star with mass M. The radius in Schwarzschild coordinates is $r = 7M$.
a.) What is the period measured by an observer at infinity?
b.) What is the period measured by a clock onboard the spaceship?

Solutions are discussed Monday, 15.02.10