## Exercise sheet 7

## Hartle 12-14.

Once across the event horizon of a black hole, what is the longest proper time an observer can spent before being destroyed in the singularity?

## Static spherically symmetric metric.

i) Write down the most general static spherically symmetric metric.

ii) Find the nine non-zero connection coefficient  $\Gamma^a_{\ bc}$  from

$$\Gamma^a{}_{bc} = \frac{1}{2}g^{ad}(\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc})$$

or

iii) use the Lagrange equations for  $L = g_{ab} \dot{x}^a \dot{x}^b$  to read off the connection coefficient from the geodesic equation.

## Scale invariance

Consider the effect of a scale transformation  $x \to e^{\alpha} x$  on a scalar field with Lagrange density

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - m^2 \phi^2 - \frac{1}{4} \lambda \phi^4$$

assuming that it acts linearly on the fields,

 $\phi(x) \to \exp(D\alpha)\phi(e^{\alpha}x).$ 

i) Write down the infinitesimal version of the scale transformation and show that  $\mathcal{L}$  is invariant, if m = 0. Determine the value of D.

ii) Find the corresponding conserved current  $s^{\mu}$ .

iii) [For the dedicated student] Show that the current  $s^{\mu}$  can be written as  $s^{\mu} = x^{\nu} \tilde{T}_{\mu\nu}$ , where  $\tilde{T}_{\mu\nu}$  is an "improved" energy-momentum tensor. Hint: Proceed similar as in the case of the angular momentum tensor.

Solutions are discussed Monday, 08.03.10