

**NTNU Trondheim, Institutt for fysikk****Examination for FY3464 Quantum Field Theory I**

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Allowed tools: mathematical tables

**1. Miscellaneous and quiz**

(Several answers could be correct.)

- a.) Write down
- $A^*$
- for (4 pts)

$$A = \bar{u}(p_2)\gamma^\mu u(p_1)$$

- b.) Calculate (4 pts)

$$\text{tr}[\gamma^\mu \gamma^\nu \gamma_\mu \gamma_\nu].$$

- c.) The covariant derivative of a Yang-Mills theory transforms under a local gauge transformation
- $U(x)$
- as: (1 pt)

- $D \rightarrow D' = D$   
  $D \rightarrow D' = U(x)D$   
  $D \rightarrow D' = U(x)DU^\dagger(x)$   
  $D \rightarrow D' = U(x)DU^\dagger(x) + \frac{i}{g}(\partial_\mu U(x))U^\dagger(x)$

- d.) The field strength of a Yang-Mills theory transforms under a local gauge transformation
- $U(x)$
- as: (1 pt)

- $\mathbf{F}(x) \rightarrow \mathbf{F}'(x) = \mathbf{F}(x)$   
  $\mathbf{F}(x) \rightarrow \mathbf{F}'(x) = U(x)\mathbf{F}(x)$   
  $\mathbf{F}(x) \rightarrow \mathbf{F}'(x) = U(x)\mathbf{F}(x)U^\dagger(x)$   
  $\mathbf{F}(x) \rightarrow \mathbf{F}'(x) = U(x)\mathbf{F}(x)U^\dagger(x) + \frac{i}{g}(\partial_\mu U(x))U^\dagger(x)$

**2. Scalar field.**Consider a real, scalar field  $\phi$  with mass  $m$  and self-interaction  $g\phi^3$ .

- a.) Write down the Lagrange density  $\mathcal{L}$ , explain your choice of signs and pre-factors (when physically relevant). (6 pts)  
b.) Write down the generating functional for connected Green function. (4 pts)  
c.) Determine the mass dimension in  $d = 4$  space-time dimensions of all quantities in the Lagrange density  $\mathcal{L}$ . (6 pts)  
d.) Draw the divergent one-loop diagrams and determine their superficial degree of divergence  $D$  (in  $d = 4$  space-time dimensions). (6 pts)

- e.) Determine the number  $d$  of space-time dimension for which the theory is renormalisable. (6 pts)

### 3. Fermion with Yukawa interaction.

Consider a fermion  $\psi$  with mass  $m$  interacting with real scalar field  $\phi$  with mass  $M$  through a Yukawa interaction,

$$\mathcal{L} = -ig\bar{\psi}\gamma^5\psi\phi.$$

- a.) Determine the global symmetries of this Lagrangian for  $m = M = 0$  and the resulting Noether charges.  
 b.) Calculate the self-energy  $\Sigma(\not{p})$  of a fermion with momentum  $p$  using dimensional regularisation. Express  $\Sigma(\not{p})$  as

$$\Sigma(\not{p}) = \frac{A}{\varepsilon} + B \ln(D/\mu^2)]. \quad (1)$$

- d.) What is your interpretation of the functional form of  $A$ ?  
 s.) What is your interpretation on the dependence of the self-energy the parameter  $\mu$ ? [max. 50 words or one formula without explicit calculation is enough] (6 pts)

### 4. Spin-1 fields.

- a.) Use the tensor method to determine the propagator  $D_{\mu\nu}(k)$  of a massive spin-1 field described by the Proca equation (6 pts)

$$(\eta^{\mu\nu}\square - \partial^\mu\partial^\nu)A_\nu + m^2A^\mu = 0.$$

- b) Give one argument why this method does not work for  $m = 0$ . (3 pts)

## Feynman rules and useful formulas

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \quad (2)$$

$$\{\gamma^\mu, \gamma^5\} = 0 \quad \text{and} \quad (\gamma^5)^2 = 1. \quad (3)$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \quad (4)$$

$$\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0 \quad (5)$$

$$\frac{1}{ab} = \int_0^1 \frac{dz}{[az + b(1-z)]^2}. \quad (6)$$

$$\frac{1}{k^2 + m^2} = \int_0^\infty ds e^{-s(k^2 + m^2)} \quad (7)$$

$$I_0(\omega, \alpha) = \int \frac{d^{2\omega} k}{(2\pi)^{2\omega}} \frac{1}{[k^2 - m^2 + i\varepsilon]^\alpha} = i \frac{(-1)^\alpha}{(4\pi)^\omega} \frac{\Gamma(\alpha - \omega)}{\Gamma(\alpha)} [m^2 - i\varepsilon]^{\omega - \alpha}. \quad (8)$$

$$I(\omega, 2) = i \frac{1}{(4\pi)^\omega} \frac{\Gamma(2 - \omega)}{\Gamma(2)} \frac{1}{[m^2 - q^2 z(1-z)]^{2-\omega}}. \quad (9)$$

$$\int_{-\infty}^{\infty} dx \exp(-x^2/2) = \sqrt{2\pi} \quad (10)$$

$$f^{-\varepsilon/2} = 1 - \frac{\varepsilon}{2} \ln f + \mathcal{O}(\varepsilon^2). \quad (11)$$

$$\Gamma(z) = \int_0^\infty dt e^{-t} t^{z-1} \quad (12)$$

$$\Gamma(n+1) = n! \quad (13)$$

$$\Gamma(-n + \varepsilon) = \frac{(-1)^n}{n!} \left[ \frac{1}{\varepsilon} + \psi_1(n+1) + \mathcal{O}(\varepsilon) \right], \quad (14)$$

$$\psi_1(n+1) = 1 + \frac{1}{2} + \dots + \frac{1}{n} - \gamma, \quad (15)$$