NTNU Trondheim, Institutt for fysikk

Examination for FY3403 Particle Physics
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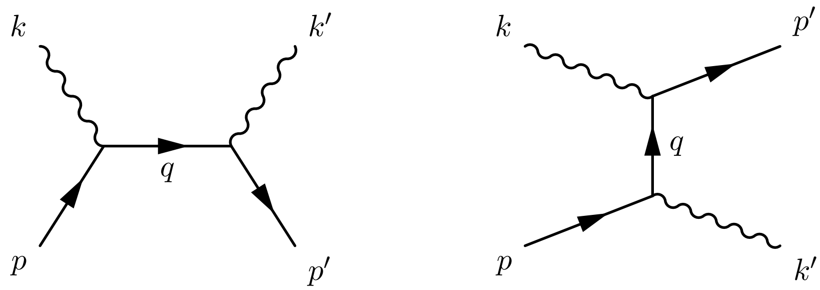
Allowed tools: mathematical tables, pocket calculator
Some formulas and data can be found on p. 2ff.

1. Interactions.
Which interaction (weak, strong, electromagnetic) is responsible for the following decays or scatterings and why:
a.) $\rho^0 \to \pi^+\pi^-$ (2 pts)
b.) $e^+e^+ \to e^+e^+$ (2 pts)
c.) $\Delta^+ \to p\pi^0$ (2 pts)

2. Feynman diagrams and amplitudes.
Assume that fermions are not mixed, i.e that $U_{\mathrm{CKM}} = U_{\mathrm{PMNS}} = 1$. Draw all tree-level diagrams (i.e. containing no loops) and write down the Feynman amplitude in momentum space as in the example given. If the process is not allowed, give the reason. (10 pts)
Example:



$$\mathrm{i}\mathcal{M} = -\mathrm{i}e^2\bar{u}(p')\left[\not{\varepsilon}^{*\prime}\,\frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}\,\not{\varepsilon} + \not{\varepsilon}\,\frac{\not{p}-\not{k}'+m}{(p-k')^2-m^2}\,\not{\varepsilon}^{*\prime}\right]u(p).$$

a.) $e^- + \nu_e \to e^- + \nu_e$:
b.) $e^- + \mu^+ \to e^- + \mu^+$
c.) $W^+ \to e^+ + \nu_e$:

3. Charged kaon decay.
Consider the decay $K^- \to l^- + \bar{\nu}_l$ with $l = \{e, \mu, \tau\}$ in the limit $m_K \ll m_W$.
a.) Parametrise the unknown coupling between a $W_\mu^\pm$ boson and the kaon by the form factor F_K^μ and write down the Feynman amplitude $\mathcal{M}$ for this decay. (4 pts)
b.) Express the form factor F_K^μ by the scalar function f_K ("the kaon decay constant") and

the relevant tensor(s). (3 pts)
c.) Calculate $|\bar{\mathcal{M}}|^2$ averaging over initial and summing over final spins. You should obtain (10 pts)

$$|\bar{\mathcal{M}}|^2 = |\bar{\mathcal{M}}|^2 = \left(\frac{g}{2m_W}\right)^4 f_K^2 m_l^2 (m_K^2 - m_l^2).$$

d.) Calculate the decay rate Γ. (10 pts)
e.) The ratio of the decay rates is

$$\frac{\Gamma(K^- \to \mu^- + \bar{\nu}_\mu)}{\Gamma(K^- \to e^- + \bar{\nu}_e)} = \frac{m_\mu^2 (m_K^2 - m_\mu^2)^2}{m_e^2 (m_K^2 - m_e^2)^2}.$$

Explain the origin of the factors. (4 pts)
f.) Explain qualitatively the effect of neutrino oscillations, if the neutrino is observed at the distance L from the decay point. (4 pts)

4. The Higgs doublet.
Consider the complex scalar SU(2) doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + \mathrm{i}\phi_2 \\ \phi_3 + \mathrm{i}\phi_4 \end{pmatrix}. \tag{1}$$

with the Lagrangian

$$\mathscr{L} = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2. \tag{2}$$

a) Show that the Lagrangian is invariant under global SU(2) and U(1) transformations. (6 pts)
b.) Write down the corresponding Lagrangian which is invariant under local SU(2) and U(1) transformations. (6 pts)
c.) Explain the difference between the two cases $\mu^2 > 0$ and $\mu^2 < 0$. (4 pts)

Useful formulas and Feynman rules

The gamma matrices form a Clifford algebra,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\,. \tag{3}$$

The matrix

$$\gamma^5 \equiv \gamma_5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3\,. \tag{4}$$

satisfies $(\gamma^5)^\dagger = \gamma^5$, $(\gamma^5)^2 = 1$, and $\{\gamma^\mu, \gamma^5\} = 0$.

$$\overline{\Gamma} \equiv \gamma^0\Gamma^\dagger\gamma^0\,. \tag{5}$$

The trace of an odd number of γ^μ matrices vanishes, as well as

$$\mathrm{tr}[\gamma^5] = \mathrm{tr}[\gamma^\mu\gamma^5] = \mathrm{tr}[\gamma^\mu\gamma^\nu\gamma^5] = 0\,. \tag{6}$$

Non-zero traces are

$$\mathrm{tr}[\gamma^\mu\gamma^\nu\ ] = 4\eta^{\mu\nu} \quad and \quad \mathrm{tr}[\not{a}\not{b}] = 4a\cdot b \tag{7}$$

$$\mathrm{tr}[\not{a}\not{b}\not{c}\not{d}] = 4[(a\cdot b)\,(c\cdot d) - (a\cdot c)\,(b\cdot d) + (a\cdot d)\,(b\cdot c)] \tag{8}$$

$$\mathrm{tr}\left[\gamma^5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\right] = 4\mathrm{i}\varepsilon_{\mu\nu\alpha\beta} \tag{9}$$

Useful are also $\not{a}\not{b} + \not{b}\not{a} = 2a\cdot b$, $\not{a}\not{a} = a^2$ and the following contractions,

$$\gamma^\mu\gamma_\mu = 4\,, \qquad \gamma^\mu\not{a}\gamma_\mu = -2\not{a}\,, \qquad \gamma^\mu\not{a}\not{b}\gamma_\mu = 4a\cdot b\,, \qquad \gamma^\mu\not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a}\,. \tag{10}$$

Completeness relations

$$\sum_s u_a(p,s)\bar{u}_b(p,s) = (\not{p}+m)_{ab}\ , \tag{11}$$

$$\sum_s v_a(p,s)\bar{v}_b(p,s) = (\not{p}-m)_{ab}\ . \tag{12}$$

Decay rate

$$\mathrm{d}\Gamma_{fi} = \frac{1}{2E_i}\,|\mathcal{M}_{fi}|^2\,\mathrm{d}\Phi^{(n)}\,. \tag{13}$$

The two particle phase space $\mathrm{d}\Phi^{(2)}$ in the rest frame of the decaying particle is

$$\mathrm{d}\Phi^{(2)} = \frac{1}{16\pi^2}\,\frac{|\boldsymbol{p}'_{\mathrm{cms}}|}{M}\,\mathrm{d}\Omega\,, \tag{14}$$

pseudoscalar mesons: $\pi^\pm$, $m = 139.6\,\mathrm{MeV}$, $(u\bar{d}), (d\bar{u})$.
$K^\pm$, $m = 493.68\,\mathrm{MeV}$, $(u\bar{s}), (s\bar{u})$.
vector mesons: ρ^0, $m = 775.5\,\mathrm{MeV}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$.
baryons: Δ^+, $m = 1232\,\mathrm{MeV}$, (uud).

	mass	energy	1/length	1/time	temperature
GeV	1.78×10^{-24} g	1.60×10^{-3} erg	5.06×10^{13} cm^{-1}	1.52×10^{24} s^{-1}	1.16×10^{13}K