Home Exam

Formalities.
Please return your solutions latest Friday, 05.10. at 12.00 either putting them into my mailbox in D5-175 or sending them by email.
If you have already your "exam number" in FY3403, use it. Otherwise put your name.
Feynman rules and standard formula you can use without derivation are appended.

Exercises.
We will use the toy theory of 3 scalar particles ϕ_i interacting with a $g\phi_A\phi_B\phi_C$ interaction presented in Ch. 6 of Griffiths to become familiar with the evaluation Feynman diagrams, cross sections and all that.

1.a) Draw the Feynman diagrams and write down the Feynman amplitude for $A + B \to A + B$ scattering at $\mathcal{O}(g^2)$. (There are 2 diagrams.) Start in coordinate space, evaluate the integrals and obtain the amplitude in momentum space.
1.b) Find the general formula for the differential cross section in the cms for $2 \to 2$ scattering.
1.c) Find the differential cross section in the cms for $A+B \to A+B$ scattering as function of the energy E_A of particle A and its scattering angle ϑ. Assume $m_A = m_B = m$ and $m_C = 0$. The ϑ dependence of your result should be

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto E_A^2 \tan^2 \vartheta/2 + m^2$$

1.d) The differential cross section diverges for $\vartheta \to 180°$ – what is your explanation for this behavior?

2.a) Draw the diagrams for $A + A \to A + A$ scattering at lowest order in g. (There are 6 diagrams; if you understood the logic in 1.a), you can draw them directly in momentum space.)
b.) Determine the amplitude for this process, setting $m_B = m_C = 0$. In your final answer, one momentum integral d^4q can remain.

The general formula for a cross section is

$$\mathrm{d}\sigma = \frac{1}{4I}\,(2\pi)^4\,\delta^{(4)}(P_i - P_f)|\mathcal{M}_{fi}|^2 \prod_{f=1}^{n} \frac{\mathrm{d}^3 p_f}{2E_f(2\pi)^3} = \frac{1}{4I}\,|\mathcal{M}_{fi}|^2\,\mathrm{d}\Phi^{(n)}\,, \tag{1}$$

where $\mathcal{M}$ is the Feynman amplitude, P_i and P_f are the total momentum of the initial and final state, $\Phi^{(n)}$ is the n-particle phase space,

$$\mathrm{d}\Phi^{(n)} = (2\pi)^4\,\delta^{(4)}(P_i - P_f)\,\prod_{f=1}^{n} \frac{\mathrm{d}^3 p_f}{2E_f(2\pi)^3}\,, \tag{2}$$

and I is the flux factor,

$$I = \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}. \tag{3}$$

(p_1 and p_2 are four-momenta of the particles in the initial state, m_1 and m_2 their masses.)

The Feynman rules (in coordinate space) are

- Draw all topologically different diagrams for the chosen order $\mathcal{O}(g^n)$ and number of external coordinates or particles.
- To each ingoing external line we associate for a scalar particle $\mathrm{e}^{-\mathrm{i}kx}$, to each outgoing external line we associate for a scalar particle $\mathrm{e}^{+\mathrm{i}kx}$.
- To each internal line connecting the points x and x' we associate a propagator $\mathrm{i}\Delta_F$ with
$$\mathrm{i}\Delta_F(x - x') = \mathrm{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{e}^{-\mathrm{i}k(x-x')}}{k^2 - m^2 + \mathrm{i}\varepsilon}.$$
(You may omit the $\mathrm{i}\varepsilon$.)
- Each vertex has a factor $-\mathrm{i}g$ and connects 3 lines, one for particle A, one for B and one for C.
- Integrate over all intermediate points.
- The Feynman amplitude $\mathrm{i}\mathcal{M}$ is given by the result omitting the delta function $(2\pi)^4\,\delta^{(4)}(P_i - P_f)$ expressing overall momentum conservation,