Formalities.
Solutions should be handed in Monday 07.10., latest 14.00, by Inspera. If Inspera doesn't work, put your solutions in my mailbox (in D5-166), sent them by email or hand them in in the lectures.

1. Scaling relation for main-sequence (MS) stars.
a.) Write down the four equations of stellar structure and the three "material relations", assuming radiative energy transfer (i.e. no convection), an ideal gas law, a power law for the energy generation rate $\propto \rho T^n$, and a constant opacity.
b.) Introduce the mass fraction $x = M(r)/M$ and characteristic quantities (i.e. characteristic radius R_*, pressure P_*, ...). Show that by introducing $r = f_1(x)R_*$, $P = f_2(x)P_*$, ..., you can split these equations into non-linear differential equations for the dimensionless functions and algebraic equations for the characteristic quantities.
c.) Derive from the algebraic equations the following scaling relations:

$$L \propto M^3 \qquad \text{and} \qquad R \propto M^{\frac{n-1}{n+3}}.$$

d.) Derive the slope of the MS in the Hertzsprung-Russel diagram for stars generating energy mainly by *i*) the pp chain ($n = 4$) and *ii*) the CNO cycle ($n = 16$).
e.) The minimmal temperature for the ignition of the pp chain is $T_{\rm min} = 4 \times 10^6\,$K. Show that the central temperature scales as $T_c \propto M^{4/(n+3)}$. Use the Sun to fix the proportionalty constant and derive the lower end of the MS.
f.) How does the expected life-time of MS stars scale?
g.) How does your results in d.) and e) compare to observations?

2. Chandrasekhar theory for white dwarf stars.
Assume that the pressure of white dwarf (WD) stars is given by completely degenerate (non-interacting) electrons. In the general case, where the relativity parameter $x = p_F/(mc)$ is neither zero or one, the E.o.S. is not a polytrope and the Lane-Emden equation has to be generalised. Chandrasekhar showed that writing the pressure and the density of degenerate electrons as function of the relativity parameter x as $P(x) = Af(x)$ and $\rho(x) = Bx^3$, one can derive

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}z}{\mathrm{d}r}\right) = -\frac{\pi G B^2}{2A}(z^2-1)^{3/2}. \qquad (1)$$

with $z^2 = x^2 + 1$.
a.) Make this equation dimensionless, introducing $x_c \equiv x(r = 0)$ and $z_c \equiv z(r = 0)$, and new variables

$$r = \alpha\eta \quad \text{and} \quad z = z_c\phi$$

satisfying

$$\alpha^2 = \frac{2A}{\pi G}\frac{1}{(Bz_c)^2} \quad \text{and} \quad z_c^2 = x_c^2 + 1$$

and show that it can be written as

$$\frac{1}{\eta^2}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\eta^2\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right) = -\left(\phi^2 - \frac{1}{z_c^2}\right)^{3/2}. \quad (2)$$

b.) Specifiy the boundary conditions.
c.) Solve this equation numerically for ten values of $1/z_0^2$ btween zero and one, find the resulting M-R relation of WDs, and plot it. Give a short interpretation: existence of minimal/maximal masses, how reliable is the result?