4.2

a) In the frame of the rocket, the lenght of the rocket is L, therefore it takes

$$\Delta t = \frac{L}{c}\,, \tag{1}$$

for the light signal to reach the nose of the rocket.

b) Eq. (4.15) in Hartle gives us the time dilitation:

$$\Delta t_{\text{earth}} = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c}}}\,, \tag{2}$$

where Δt_{earth} is the measured time between the two events in the frame of the earth. As $v = \frac{4c}{5}$, we get

$$\Delta t_{\text{earth}} = \Delta t \frac{1}{\sqrt{1 - \frac{4^2}{5^2}}} = \frac{5}{3}\frac{L}{c} \tag{3}$$

4.3

We first discuss this excercise quantitativly. To find the value of γ we use the equation for lenght contraction (Eq 4.26 in Hartle):

$$\frac{L_*}{L} = \gamma = 2\,. \tag{4}$$

Where L_* is the resting lenght (of either the pole or the barn) and L is the lenght measured if it is moving (by an observer).

The speed of the runner is then ($c = 1$)

$$\frac{1}{1 - v^2} = \gamma^2 = 4\,, \implies v = \frac{\sqrt{3}}{2}\,. \tag{5}$$

We now turn to the resolution of the appartent paradox. In the frame of the barn the two spacetime events: the front door opens E_1 and the back door closes E_2, can be parameterized by

$$E_1 = (0,\, 2d)\,, \quad E_2 = (0,\, 0)\,, \tag{6}$$

where $d = 5$m. The two events happen at the same time, and is separated by the spatial distance $2d$. We transform these events to the frame of the runner, by use of the tranformation laws given in Eqs. 4.33 in Hartle:

$$E_1' = (-\gamma v d, 4d)\,, \quad E_2' = (0, 0)\,. \tag{7}$$

In this frame the spatial distance between the events has increased, but the front door opens before the back door closes. How far does the runner (or actually the barn, the runner is at rest) move, in the frame of runner, in the time elapsing between the two events?

$$\Delta s = v(\Delta t) = v(\gamma v d) = 3d \tag{8}$$

This distance is 15m, the barn is 5m in this frame: in total 20m; the lenght of the pole.

The discussion in terms of spacetime diagrams is shown in Fig. 2.