7.2

Given the geometry specified by

$$ds^2 = -dt^2 + 2dxdt + dy^2 + dz^2\,, \tag{1}$$

we want to find a coordinate transformation which turns the metric into a usual flat space form. It is obvious that we should only transform x and t. Taking a close look at the the above expression, it is natural to perform the following rewriting

$$ds^2 = -\left(dt - dx\right)^2 + dx^2 + dy^2 + dz^2\,. \tag{2}$$

It is now obvious that $t \to t + x$ transforms the metric to the usual flat space form.

7.5

a) The light cone is given by $ds^2 = 0$:

$$ds^2 = -x dv^2 + 2 dv dx = 0 \tag{3}$$

Assuming that $dv \neq 0$, that is, time changes, we get

$$2\frac{dx}{x} = dv \qquad \Longrightarrow \qquad v = v_0 e^{x^2} \tag{4}$$

where v_0 is the value of v when $x = 0$. This is the equation for the light cone.

c) A particle can only travel on a timelike cone, so $ds^2 < 0$, also as time increases $dv > 0$:

$$ds^2 = -x dv^2 + 2 dv dx < 0 \qquad \Longrightarrow \qquad x > 2\frac{dx}{dv} \tag{5}$$

For positive x, we see that x can both decrease and increase. For negative x, however, it must decrease. This means that a particle starting at positive x can go to negative x, but not the other way around.

7.10

We deal with the metric

$$ds^2 = -d\tau^2 = -X^2 dT^2 + dX^2\,. \tag{6}$$

The four velocity is given by

$$u = \frac{dx}{d\tau}\,. \tag{7}$$

The particle is moving on the curve given by $X = 2T$ for $T > 1$. This means that

$$d\tau^2 = 4\left(T^2 - 1\right) dT^2\,, \tag{8}$$

and the curve is timelike as $d\tau > 0$.

Using the equation for the curve

$$d\tau^2 = 4(T^2 - 1) dT^2\,, \tag{9}$$