1. $\langle E_e \rangle$ in μ decay

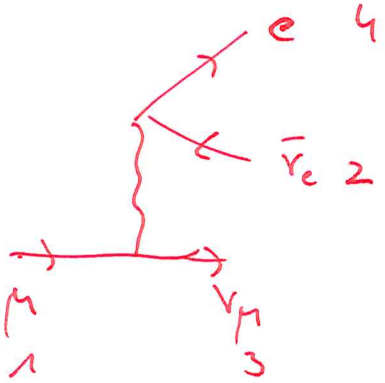
(9.33) of Griffiths ;

$$\frac{d\Gamma}{dE} = \left(\frac{g}{m_W}\right)^4 \frac{m_\mu^2 E^2}{2(4\pi)^3}\left(1 - \frac{4E}{3m_\mu}\right) \quad ; \quad M = \tfrac{1}{2} m_\mu$$

$$\langle E \rangle = \frac{\int_0^M dE\, E\, d\Gamma/dE}{\int_0^M dE\, d\Gamma/dE} = \frac{\int_0^M dE\, E^3\left(1 - \frac{2E}{3M}\right)}{\int_0^M dE\, E^2\left(1 - 2E/3M\right)}$$

$$= \ldots = \frac{7}{20} m_\mu$$

2. μ decay with pure V interaction



$$-i\frac{g}{2\sqrt{2}}\gamma^\mu$$

$$M = \frac{g^2}{8 m_W^2}\left[\bar{u}(3)\gamma^\mu u(1)\right]\left[\bar{u}(4)\gamma_\mu \bar{v}(2)\right]$$

$$\langle |\bar{M}|^2 \rangle = \frac{1}{4}\left(\frac{g}{m_W}\right)^4 \left[(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right]$$

rest-frame of μ:

$$p_1\cdot p_2 = m_\mu E_2 \qquad p_1\cdot p_4 = m_\mu E_4$$

for $p_3\cdot p_4$: $\quad (p_3+p_4)^2 = m_e^2 + 2\,p_3\cdot p_4$

$$= (p_1 - p_2)^2 = m_\mu^2 - 2p_1\cdot p_2 = m_\mu^2 - 2m_\mu E_2$$

$$\Rightarrow \quad p_3\cdot p_4 = \frac{m_\mu^2 - m_e^2}{2} - m_\mu E_2$$

similarly $\quad p_2\cdot p_3 = \dfrac{m_\mu^2 - m_e^2}{2} - m_\mu E_4$

set $m_e = 0$: $\quad m_\mu = m$

$$\langle |M|^2 \rangle = \frac{1}{4}\left(\frac{g}{m_W}\right)^4 \left[m^2 E_2\left(\tfrac{1}{2}m_\mu - E_2\right) + m^2 E_4\left(\tfrac{1}{2}m - E_4\right)\right]$$

$|\vec{p}_2| = E_2$ etc: $\quad = \frac{1}{8}\left(\frac{g}{m_W}\right)^4 m^2 \underbrace{\left[p_2(m_\mu - 2p_2) + p_4(m_\mu - 2p_4)\right]}$

result for V-A

copying result for $d\rho_2$ integral

$$\Gamma_{(p_2)} = \left(\frac{g}{4\pi m_U}\right)^4 m_\mu \left(\frac{m_\mu}{2} - \frac{2}{3} p_4\right) d^3 p_4$$

for a new combination (9.31):

$$d\Gamma = \frac{\langle |M|^2 \rangle}{(4\pi)^4 m_\mu} d|\vec{p}_2| \frac{d^3 p_4}{|\vec{p}_4|^2}$$

$$d\Gamma_{(p_4)} = \frac{1}{8}\left(\frac{g}{4\pi m_U}\right)^4 m_\mu \frac{d^3 p_4}{|\vec{p}_4|^2} p_4 (m_\mu - 2p_4) \int_{p_4}^{\frac{1}{2}m_\mu} d|\vec{p}_2|$$

$\underbrace{\frac{1}{2}m_\mu - p_4}$

combining both terms & $d^3 p_4 = 4\pi |\vec{p}_4|^2 d|\vec{p}_4|$

& $E = |\vec{p}_4|$

$$\frac{d\Gamma}{dE} \propto E^2\left(1 - \frac{16E}{9 m_\mu}\right)$$