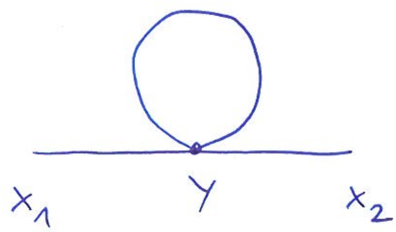


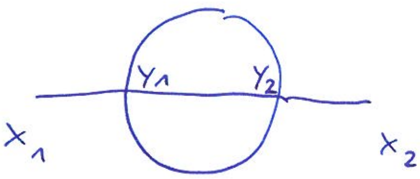
first order in λ

3 remaining possibilities

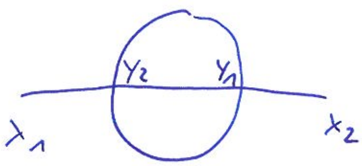
$$-\frac{i\lambda}{4!}\int d^4y \, \langle 0|T[\phi(x_1)\,\phi(x_2)\,\phi(y)\,\phi(y)\,\phi(y)\,\phi(y)]|0\rangle$$

4

$$S = \frac{4\cdot 3}{4!} = \frac{1}{2}$$



+



since y_1, y_2 are integration variable
$\Rightarrow$ factor 2

second order:

$$\frac{1}{2!}\left(-\frac{i\lambda}{4!}\right)^2 \int d^4y_1\, d^4y_2 \, \langle 0|T[\phi(x_1)\phi(x_2)\,\phi(y_1)\,\phi(y_1)\,\phi(y_1)\,\phi(y_1)\,\phi(y_2)\,\phi(y_2)\,\phi(y_2)\,\phi(y_2)]|0\rangle$$

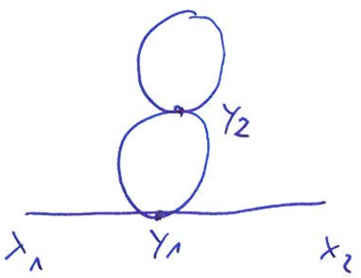
$$+\ (y_1 \leftrightarrow y_2)$$

$\phi(x_1)\ \phi(y_1)$: 4 ways

$\phi(x_2)\ \phi(y_2)$: 4 ways

3! ways to contract remaining pairs of $\phi(y_1)$ and $\phi(y_2)$

$$\Rightarrow \quad S = \left(\frac{1}{2!}\cdot 2\right)\left(\frac{1}{4!}\right)^2 4\cdot 4\cdot 3! = \frac{4\cdot 4\cdot 3\cdot 2}{4!\ 4\cdot 3\cdot 2} = \frac{1}{3!}$$



again second order

$\phi(x_1)\ \phi(y_1)$: 4 ways

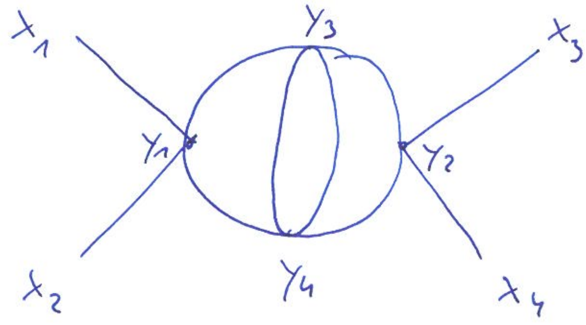
remaining: $\phi(x_2)\ \phi(y_1)$: 3 ways

a $\phi(y_1)\phi(y_1)$ pair left

$\phi(y_2)\phi(y_2)\phi(y_2)\phi(y_2)$

$\Big\}\ \binom{4}{2} = 4\cdot 3$ ways

$$\Rightarrow \quad S = \left(\frac{1}{2!}\,2\right)\left(\frac{1}{4!}\right)^2 4\cdot 3\cdot 4\cdot 3 = \frac{1}{4}$$



$S = \left(\frac{1}{4!}\,\frac{4!}{2}\right)\left(\frac{1}{4!}\right)^4 \cdot (4\cdot 3)^2$

$\times (2\cdot 4\cdot 4) \times (2\cdot 3\cdot 3) \times 2$

$= \frac{1}{4}$

4. order: for fixed $x_1x_2x_3x_4$ there 4! ways to permute $y_1 \cdots y_4$ positions

identical

$\Rightarrow$ $\frac{4!}{2}$ topologically different diagrams vers $\frac{1}{4!}$ from Taylor

$x_1x_2x_3x_4$ $y_1y_1y_1y_1$ $y_2y_2y_2y_2$ $y_3y_3y_3y_3$ $y_4y_4y_4y_4$

$4\cdot 3$ ways to contract $\phi(x_1), \phi(x_2)$ with $4\,\phi(y_1)$

$4\cdot 3$ $\phi(x_3), \phi(x_4)$ $4\,\phi(y_2)$

remaining $2\,\phi(y_1)$ with $4\,\phi(y_3)$ & $4\,\phi(y_4)$: $2\cdot 4\cdot 4$

$2\,\phi(y_2)$ $2\cdot 3\cdot 3$

left $2\,\phi(y_3)$ & $2\,\phi(y_4)$: 2 ways