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Løsning Øving 3 a)

A) Tiden fra 1 til 2: $t_{12} = \int_1^2 \frac{ds}{v}$ skal ha minimum.

Energibevarelse gir $v = \sqrt{2gx} \Rightarrow$

$$t_{12} = \int_1^2 \sqrt{\frac{dx^2 + dy^2}{2gx}} = \int_1^2 dx \sqrt{\frac{1+y'^2}{2gx}}$$

Definerer $I = \int_1^2 \sqrt{\frac{1+y'^2}{x}}\, dx \equiv \int_1^2 f\, dx$

Euler: $\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0 \Rightarrow$

$$\frac{d}{dx}\left[\frac{y'}{x}\sqrt{\frac{x}{1+y'^2}}\right] = 0$$

$$\frac{y'}{x}\sqrt{\frac{x}{1+y'^2}} = \text{konst} = \frac{1}{\sqrt{2k}}$$

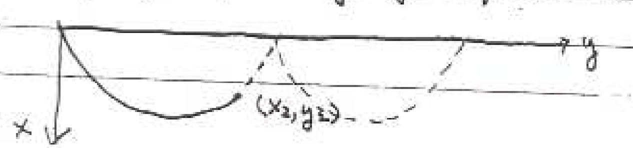
Kvadrering $\Rightarrow \frac{y'^2}{x(1+y'^2)} = \frac{1}{2k}$, $y = \int_1^2 \frac{x\,dx}{\sqrt{2kx - x^2}}$

Parameter θ defineres ved $\underline{x = k(1-\cos\theta)}$, $dx = k\sin\theta d\theta$

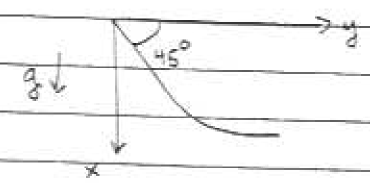
$\Rightarrow$

$$y = \int_1^2 k(1-\cos\theta)\,d\theta = \underline{k(\theta - \sin\theta)}$$

Ligningene for x og y definerer en sykloide:



Løsning Oppgave II b) (Øv. 3 b))



a) Velger nullnivået for potensiell energi ved $x = 0$.

Energibevarelse: $\frac{1}{2}v_0^2 = \frac{1}{2}v^2 - gx$,

$v^2 = v_0^2 + 2gx$.

Brakistokronkurven bestemt ved

$$\int \frac{ds}{v} = \int \frac{\sqrt{1+y'^2}}{\sqrt{v_0^2+2gx}}\,dx \quad \text{er minimum.}$$

Definerer $f = \frac{\sqrt{1+y'^2}}{\sqrt{v_0^2+2gx}}$

Euler:

$$\underbrace{\frac{\partial f}{\partial y}}_{0} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$$

$$\frac{d}{dx}\Bigg[\underbrace{\frac{1}{\sqrt{v_0^2+2gx}}\cdot\frac{y'}{\sqrt{1+y'^2}}}_{C}\Bigg] = 0$$

$$y'^2 = C^2(v_0^2+2gx)(1+y'^2)$$

$$y'^2 = \frac{C^2(v_0^2+2gx)}{1 - C^2(v_0^2+2gx)}$$

Utgangsbetingelsen i origo er $y'(0) = 1$, $\Rightarrow$

$$1 = \frac{C^2v_0^2}{1 - C^2v_0^2}, \quad C^2 = \frac{1}{2v_0^2} \quad \text{Dermed blir}$$

$$y'^2 = \frac{\frac{1}{2v_0^2}(v_0^2+2gx)}{1 - \frac{1}{2v_0^2}(v_0^2+2gx)} = \underline{\frac{h_0+x}{h_0-x}}, \quad \underline{h_0 = \frac{v_0^2}{2g}}$$