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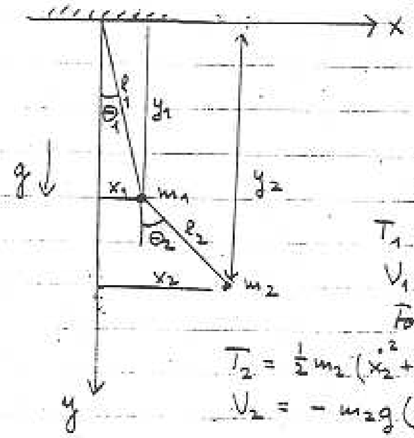
Løsning Øving 5



a) Av figuren er

$x_1 = l_1 \sin\theta_1$, $y_1 = l_1\cos\theta_1$

$x_2 = l_1\sin\theta_1 + l_2\sin\theta_2$, $y_2 = l_1\cos\theta_1 + l_2\cos\theta_2$

For masse m_1:

$T_1 = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^2$

$V_1 = -m_1 g l_1 \cos\theta_1$.

For masse m_2:

$T_2 = \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) = \frac{1}{2}m_2\left[l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2l_1l_2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2\right]$

$V_2 = -m_2 g(l_1\cos\theta_1 + l_2\cos\theta_2)$.

Lagrangefunksjon $L = T_1 + T_2 - V_1 - V_2 =$

$= \frac{1}{2}(m_1+m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^2 + m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2) +$

$+ (m_1+m_2)g l_1\cos\theta_1 + m_2 g l_2\cos\theta_2$.

Lagrange:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}_i} - \frac{\partial L}{\partial\theta_i} = 0, \quad i = 1,2, \quad \Rightarrow$$

$$(m_1+m_2)l_1^2\ddot{\theta}_1 + m_2 l_1 l_2\ddot{\theta}_2\cos(\theta_1-\theta_2) + m_2 l_1 l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) + (m_1+m_2)g l_1\sin\theta_1 = 0$$

$$m_2 l_2\ddot{\theta}_2 + m_2 l_1 l_2\ddot{\theta}_1\cos(\theta_1-\theta_2) - m_2 l_1 l_2\dot{\theta}_1^2\sin(\theta_1-\theta_2) + m_2 g l_2\sin\theta_2 = 0.$$

b) $m_1 = m_2 = m$, $l_1 = l_2 = l$ gir

$$2\ddot{\theta}_1 + \ddot{\theta}_2\cos(\theta_1-\theta_2) + \dot{\theta}_2^2\sin(\theta_1-\theta_2) + 2\frac{g}{l}\sin\theta_1 = 0$$

$$\ddot{\theta}_2 + \ddot{\theta}_1\cos(\theta_1-\theta_2) - \dot{\theta}_1^2\sin(\theta_1-\theta_2) + \frac{g}{l}\sin\theta_2 = 0$$

Løsning Øv. 5, forts.

Små utslag, $\theta_i \ll 1$, gir til laveste orden ettersom $\sin\theta_i \simeq \theta_i$, $\cos\theta_i \simeq 1$,

$$2\ddot{\theta}_1 + \ddot{\theta}_2 + 2\omega_0^2\theta_1 = 0 \qquad \omega_0 = \sqrt{\frac{g}{l}}$$

$$\ddot{\theta}_2 + \ddot{\theta}_1 + \omega_0^2\theta_2 = 0$$

Ansats $\theta_i = A_i\cos\omega t \Rightarrow$

$$(2\omega_0^2 - 2\omega^2)A_1 - \omega^2 A_2 = 0$$

$$-\omega^2 A_1 + (\omega_0^2 - \omega^2)A_2 = 0$$

Betingelsen for løsning er at determinanten forsvinner:

$$\begin{vmatrix} 2\omega_0^2 - 2\omega^2, & -\omega^2 \\ -\omega^2, & \omega_0^2 - \omega^2 \end{vmatrix} = 0.$$

Altså

$$\omega^4 - 4\omega_0^2\omega^2 + 2\omega_0^4 = 0.$$ Løsningene er

$$\omega_1^2 = (2+\sqrt{2})\omega_0^2, \qquad \omega_2^2 = (2-\sqrt{2})\omega_0^2.$$