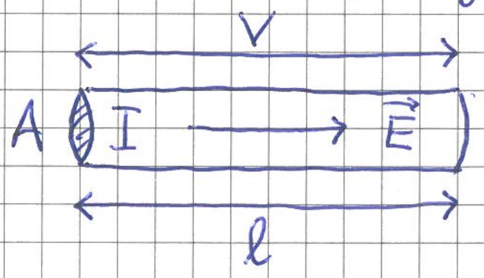
En <u>motstand</u> er en (typisk <u>dårlig</u>) leder med tverrsnitt A, lengde ℓ:



$$V = E\cdot \ell, \quad j = \frac{I}{A} = \sigma E = \sigma \frac{V}{\ell}$$

$$\Rightarrow I = \frac{\sigma A}{\ell}\cdot V = G\cdot V$$

$$V = \frac{\ell}{\sigma A}\cdot I = R\cdot I$$

med

$G = \frac{\sigma A}{\ell}$ = lederens <u>konduktans</u>

$R = G^{-1} = \frac{\ell}{\sigma A}$ = lederens <u>resistans</u> ("motstand")

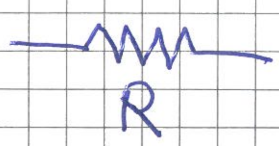
$\rho = \sigma^{-1}$ = materialets <u>resistivitet</u>

Enheter:

$$[R] = \frac{V}{A} = \Omega \text{ (ohm)} \Rightarrow [G] = \Omega^{-1}$$

$$\Rightarrow [\rho] = \Omega m \Rightarrow [\sigma] = \Omega^{-1} m^{-1}$$

σ og ρ er <u>materialspesifikke</u>, G og R er også avhengig av lederens <u>dimensjoner</u>

Kretssymbol:  eller 

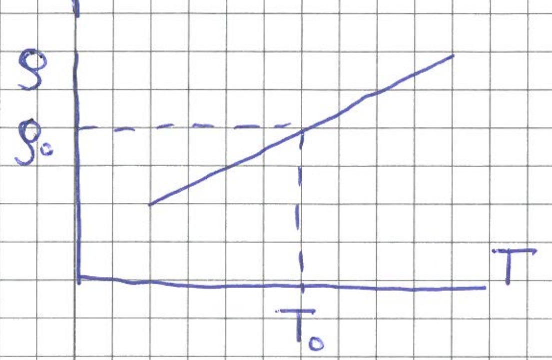
$\rho(T)$ [YF 25.2; LHL 21.2, 21.5]

[Drude: $\rho = 1/\sigma = m_e/ne^2\tau$]

Metaller

Større T $\Rightarrow$ Flere kollisjoner $\Rightarrow$ Større ρ

Empirisk:



$$\rho(T) = \rho_0\left[1 + \alpha(T - T_0)\right]$$

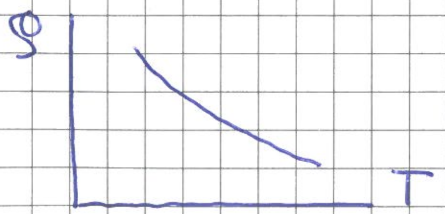
$$\alpha_{Al} \approx \alpha_{Cu} \approx \alpha_{Ag} \approx 0.004\ K^{-1}$$

$([T] = K \text{ (kelvin)})$

(Dus: $\tau \sim 1/T$)

Halvledere (Si, Ge, GaAs, ...)

- Isolator ved $T = 0$
- Større T $\Rightarrow$ Flere frie ladninger $\Rightarrow$ Redusert ρ



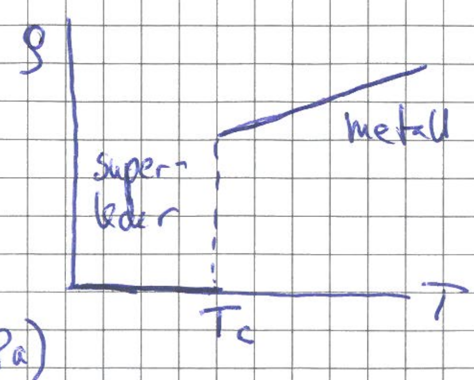
$\tau \sim 1/T$, $n \sim e^{-T_g/T}$

$\Rightarrow \rho \sim T\, e^{T_g/T}$

Si: $k_B T_g \approx 0.55\ eV$; $T_g \approx 6400\ K$

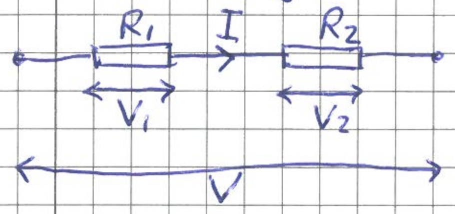
Superledere

- $\rho = 0$ for $T < T_c$ = kritisk temp.
- metall for $T > T_c$
- 1911: Hg, $T_c = 4.12\ K$
- 2015: H_3S (?), $T_c = 203\ K$ ($p = 150\ GPa$)



Kobling av flere motstander [YF 26.1; LHL 21.3]

Seriekobling:



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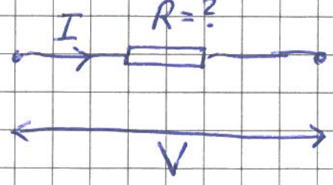
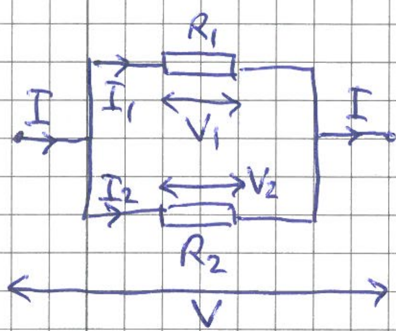


$$\Rightarrow \quad V = V_1 + V_2 = R_1 I + R_2 I = (R_1 + R_2) I \quad ; \qquad V = RI$$

$$\Rightarrow \quad \boxed{R = R_1 + R_2} \qquad N \text{ i serie: } \boxed{R = \sum_{j=1}^{N} R_j}$$

Parallellkobling:



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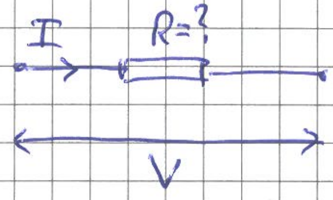


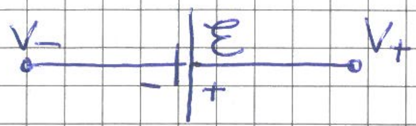
$$\Rightarrow \quad I = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad ; \qquad I = V \cdot \frac{1}{R}$$

$$\Rightarrow \quad \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}} \qquad N \text{ i parallell: } \boxed{R^{-1} = \sum_{j=1}^{N} R_j^{-1}}$$

DC - kretser [YF 26 (25) ; LGC 22]

(DC = direct current = likestrøm)

Likespenningskilde:



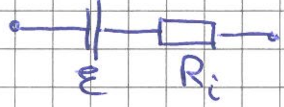
Sørger for spenning (= potensialforskjell) $\mathcal{E} = V_+ - V_-$ mellom polene

$\mathcal{E}$ = elektromotorisk spenning (ems)

= tilført energi pr ladningsenhet

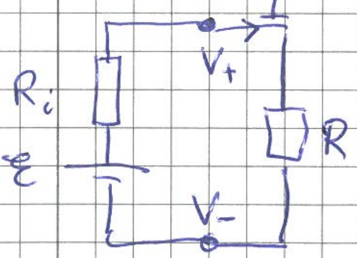
Eksempler: kjemisk batteri, solcelle, ...

Reell kilde:



R_i = indre motstand i spenningskilden

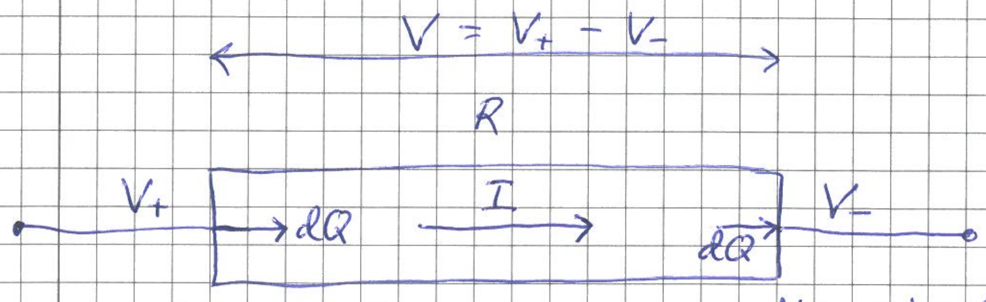
Ideell kilde: $R_i = 0$



$$\mathcal{E} = R_i I + R I$$

$V_+ - V_- = \mathcal{E} - R_i I$ = spenning levert av reell kilde ($< \mathcal{E}$ når $I > 0$)

Elektrisk effekt [YF 25.5 ; LHL 22.2]



$dU_{inn} = V_+ \, dQ$ $\qquad$ $dU_{ut} = V_- \, dQ$

Effekttap i motstanden: [El. energi → Varme]

$$P = \frac{dU}{dt} = \frac{dU_{inn} - dU_{ut}}{dt} = \frac{V_+ dQ - V_- dQ}{dt} = V \frac{dQ}{dt} = \underline{\underline{VI}}$$

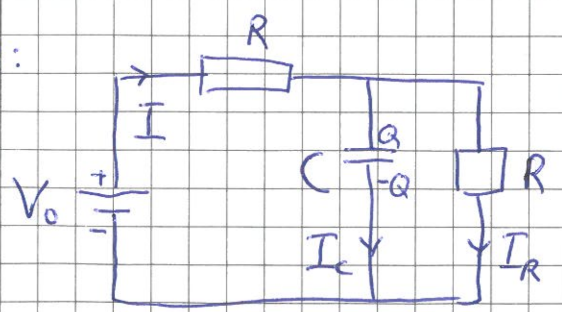
Med ohmsk motstand: $P = VI = RI^2 = V^2/R$

Kirchhoffs regler [YF 26.2 ; LHL 22.3]

Pga ladningsbevarelse: $\boxed{\sum_j I_j = 0 \text{ i alle knutepunkt}}$ ("K1")

Pga energibevarelse: $\boxed{\sum \text{potensialendringer} = 0 \text{ for alle sløyfer}}$ ("K2")

Eks:



$K1 \Rightarrow I = I_C + I_R$

$K2 \Rightarrow V_0 - RI - Q/C = 0$

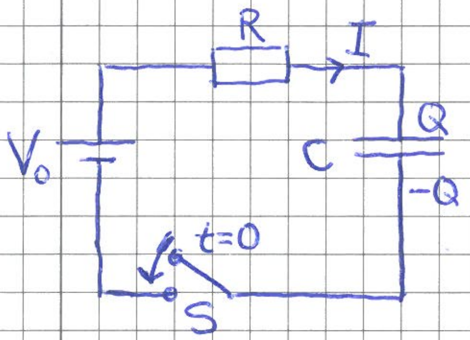
$K2 \Rightarrow V_0 - RI - RI_R = 0$

RC-krets [YF 26.4 ; LHL 22.4]

$$V = RI$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} \; ; \quad I = \frac{dQ}{dt}$$

Opplading av kondensator i RC-krets:



- $Q(0) = 0$
- S lukkes ved $t = 0$
- Finn $Q(t)$ og $I(t)$

$$K2 \Rightarrow V_0 - RI - \frac{Q}{C} = 0$$

$$\Rightarrow R\frac{dQ}{dt} = -\frac{Q}{C} + V_0 = -\frac{Q - V_0 C}{C}$$

$$\Rightarrow \int_0^Q \frac{dQ}{Q - V_0 C} = -\int_0^t \frac{dt}{RC}$$

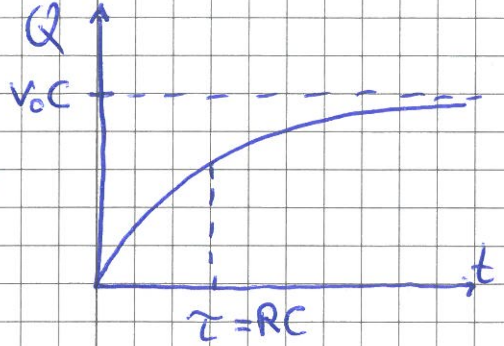
$$\Rightarrow \ln\left[\frac{Q - V_0 C}{-V_0 C}\right] = -\frac{t}{RC}$$

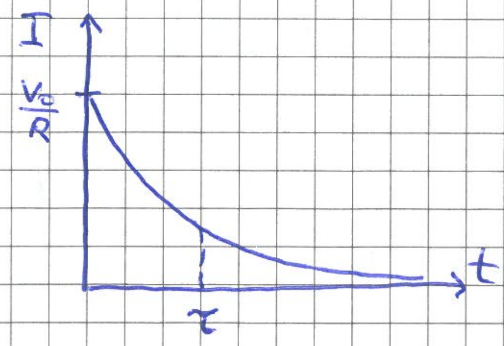
$$\Rightarrow \underline{Q(t) = V_0 C \left\{1 - e^{-t/RC}\right\}}$$

$$\Rightarrow \underline{I(t) = \frac{dQ}{dt} = \frac{V_0}{R} e^{-t/RC}}$$

Mekanisk analogi: $F_0 - b\dot{x} - kx = m\ddot{x} \xrightarrow{m \to 0} 0$

$$\Rightarrow x(t) = \frac{F_0}{k}\left\{1 - e^{-kt/b}\right\} ; \quad \dot{x}(t) = \frac{F_0}{b} e^{-kt/b}$$

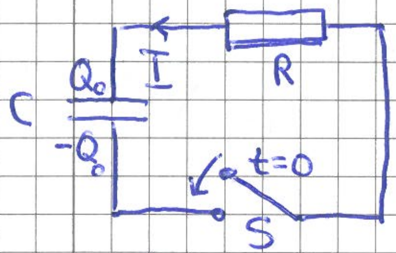




$\tau = RC$ = kretsens <u>tidskonstant</u> = karakteristisk tid for oppladning og utladning av kondensator i RC-krets.

$$Q(t=3\tau) = V_0 C\left(1-e^{-3}\right) \approx 0.95\, V_0 C$$

Utlading av kondensator i RC-krets:

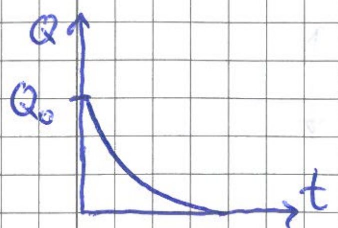


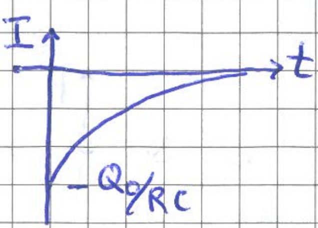
- $Q(0) = Q_0$
- S lukkes ved $t = 0$
- Finn $Q(t)$ og $I(t)$

$$K2 \Rightarrow -\frac{Q}{C} - R\frac{dQ}{dt} = 0$$

$$\Rightarrow \int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC}$$

$$\Rightarrow \underline{Q(t) = Q_0 e^{-t/RC}} \; ; \; \underline{I(t) = -\frac{Q_0}{RC} e^{-t/RC}}$$

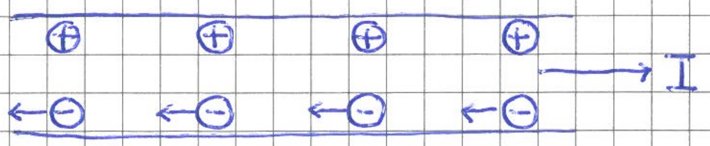




[Råd: Bruk alltid $I = +\frac{dQ}{dt}$. Får minustegn i $I(t)$ hvis vi velger feil retning på I.]

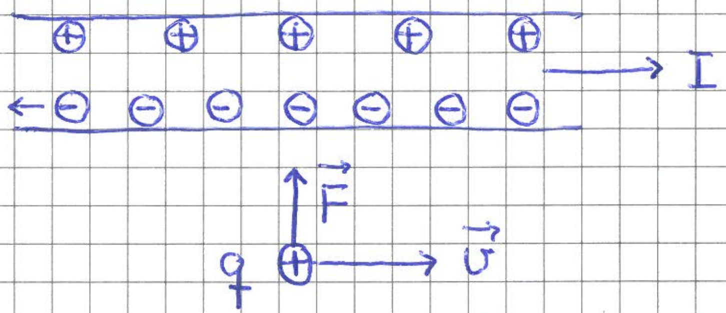
Magnetostatikk [YF 27, 28 ; LHL 23]

Coulombs lov og Einsteins relativitetsteori <u>nødvendiggjør</u> magnetfelt og magnetisk kraft:



q_0 ⊕ (i ro)

q_0 ser <u>nøytral</u> strømførende leder $\Rightarrow F = 0$



q ser <u>negativt ladet</u> strømførende leder, fordi ⊖ har større relativ hastighet enn ⊕, dermed størst <u>lengdereduksjon</u> for avstanden mellom ⊖

(Einstein: $x = x_0 \sqrt{1 - \frac{u^2}{c^2}}$; u = relativ hastighet, c = lyshastigheten)

$\Rightarrow$ q merker <u>elektrisk</u> kraft $\vec{F}$

<u>Vi</u>, som er i ro relativt lederen, kaller dette en <u>magnetisk</u> kraft $\vec{F}_m$, som uttrykkes via et <u>magnetfelt</u> $\vec{B}$, som skapes av strømmen I

Magnetisk kraft [YF 27.2; LHL 23.4]

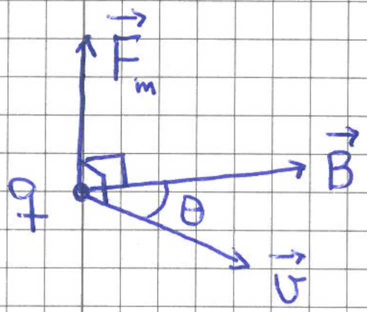
En ladning omgir seg med elektrisk felt $\vec{E}$

$\Rightarrow$ elektrisk kraft $\vec{F}_e = q\vec{E}$ på (en annen) ladning q

En strøm omgir seg med magnetfelt $\vec{B}$

$\Rightarrow$ magnetisk kraft $\vec{F}_m = q\vec{v} \times \vec{B}$ på

(en annen) ladning q i bevegelse med hastighet $\vec{v}$



$$F_m = q v B \sin\theta$$

$$\vec{F}_m \perp \vec{B} \quad \text{og} \quad \vec{F}_m \perp \vec{v}$$

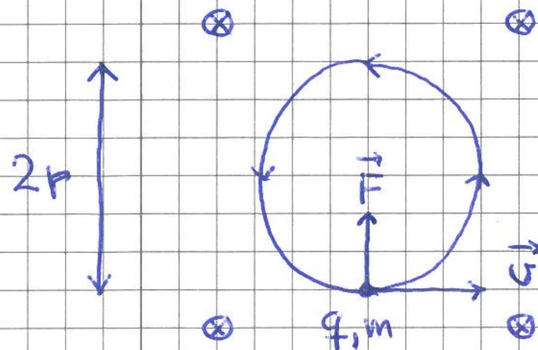
Enhet: $[B] = \left[\frac{F}{qv}\right] = \frac{N}{C \cdot m/s} = \frac{N}{A \cdot m} = T$ (tesla)

Hvis både $\vec{E}$ og $\vec{B}$ til stede der q er:

$$\boxed{\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q\vec{v} \times \vec{B}}$$

Lorentzkraften

Ladning i uniformt magnetfelt [YF 27.4; LHL 23.1, 23.4]



$\vec{v} \perp \vec{B} \Rightarrow F = qvB$

$\vec{F} \perp \vec{v} \Rightarrow$ Tilført effekt $P = \vec{F} \cdot \vec{v} = 0$

$\Rightarrow K = \frac{1}{2} m v^2 =$ konstant

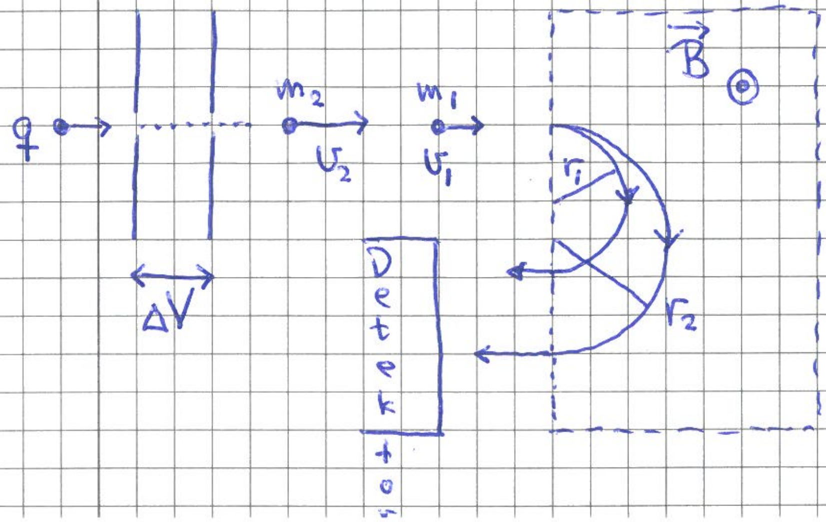
$\Rightarrow$ Sirkelbevegelse med konstant v

(uniform sirkelbevegelse)

N2: $qvB = mv^2/r \Rightarrow \boxed{r = \frac{mv}{qB}}$

Syklotronfrekvensen: $\boxed{\omega_c = \frac{v}{r} = qB/m}$

Eks: Massespektrometer (Øv. 12)

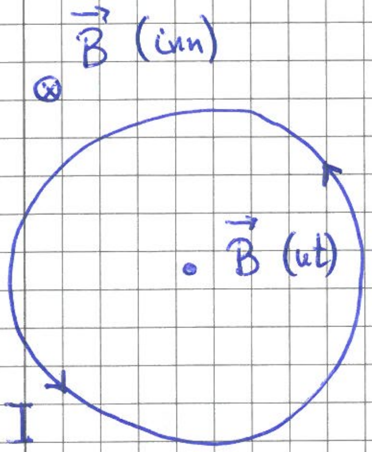


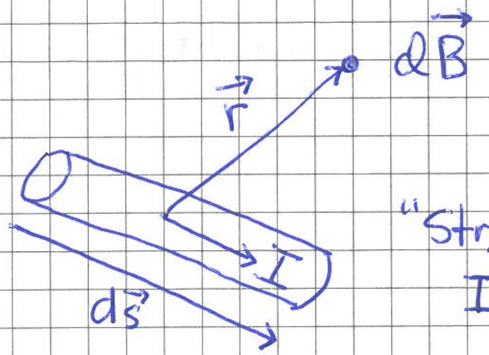
$v_j \sim \frac{1}{\sqrt{m_j}}$

$\Downarrow$

$r_j \sim \sqrt{m_j}$

Biot-Savarts lov [YF 28.2; LHL 23.5]





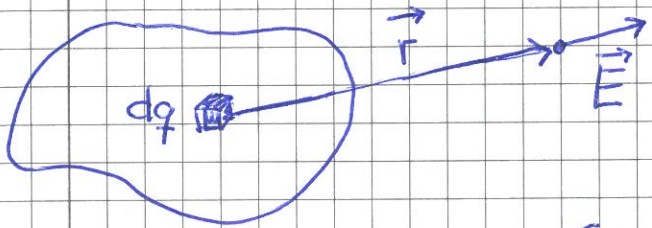
"Strømelement"
$I\,d\vec{s}$

$$\boxed{\vec{B} = \oint d\vec{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{s} \times \hat{r}}{r^2}}$$

Biot og Savart
(1820)
(Empirisk lov)

$\mu_0 = 4\pi \cdot 10^{-7} \; \frac{T \cdot m}{A}$ = vakuumpermeabiliteten

Sammenlign med Coulombs lov:



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq\,\hat{r}}{r^2}$$

$\varepsilon_0 \approx 8.85 \cdot 10^{-12} \; C^2/m^2N$
= vakuumpermittiviteten

$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ = farten til elektromagnetiske bølger i vakuum
(lysfarten)

$\approx 3 \cdot 10^8$ m/s