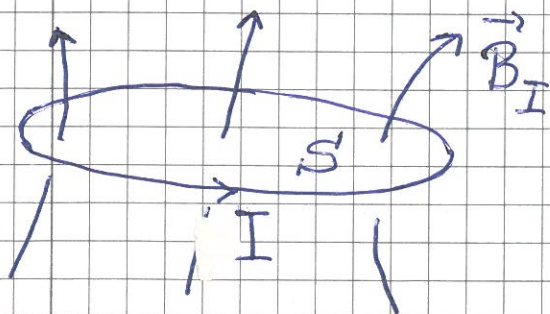


Induktans [YF 30.2; LHL 25.1]

Selvinduktans:



S = omsluttet flate

$$\Phi = \int_S \vec{B}_I \cdot d\vec{A} = \int_S \left\{ \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{s} \times \hat{r}}{r^2} \right\} \cdot d\vec{A}$$

$$= I \cdot \underbrace{\int_S \left\{ \frac{\mu_0}{4\pi} \oint \frac{d\vec{s} \times \hat{r}}{r^2} \right\} \cdot d\vec{A}}_{\text{støyfas selvinduktans } L}$$

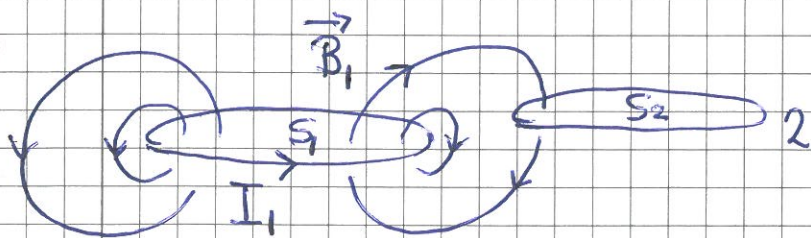
= støyfas selvinduktans L

Ser at L avhenger av utforming og medium
(her: vakuum (μ_0))

$$L = \Phi / I$$

Enhet: $[L] = T \cdot m^2 / A = H$ (henry)

Gjensidig induktans:



Strøm I_1 i sløyfe 1 skaper magnetfelt \vec{B}_1 , og dermed fluks $\Phi_2 = \int_{S_2} \vec{B}_1 \cdot d\vec{A}$ omsluttet av sløyfe 2.

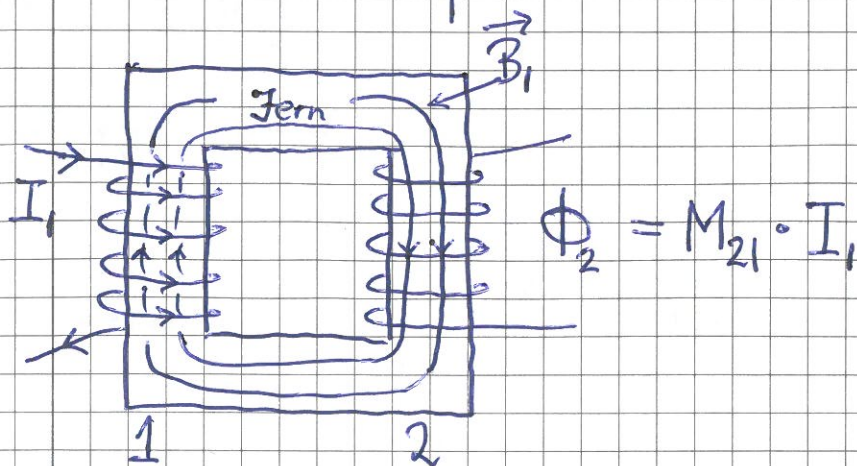
\vec{B}_1 er prop. med $I_1 \Rightarrow \Phi_2$ er også prop. med I_1

$$M_{21} = \frac{\Phi_2}{I_1} = \text{de to sløyfenes gjensidige induktans}$$

Tilsvarende: Strøm I_2 i nr 2 \Rightarrow felt $\vec{B}_2 \sim I_2$, og dermed fluks $\Phi_1 \sim I_2$ omsluttet av nr 1

$$M_{12} = \frac{\Phi_1}{I_2}; \quad M_{12} = M_{21}$$

Anvendelse: Transformator



(Selv-) Induksjon:

$$\frac{dI}{dt} \neq 0 \Rightarrow \frac{d\Phi}{dt} \neq 0 \Rightarrow \text{indusert spenning } \Delta V = -\dot{\Phi} = -L\dot{I}$$

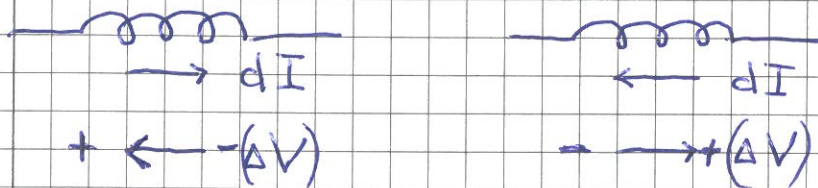
[Gjensidig induksjon: $I_1 \neq 0 \Rightarrow \dot{\Phi}_2 \neq 0 \Rightarrow \Delta V_2 = -\dot{\Phi}_2 = -M_{21}\dot{I}_1$]

Induktans som kretselement:



$\Delta V = -L \frac{dI}{dt}$

Lenz' lov gir retningen på ΔV :



Eks: Finn L for (lang, tettviklet) spole med 1000 viklinger på lengde 25 cm og tverrsnitt 10 cm^2 , luftfylt, og med jernkjerne, der $\mu_r = 2000$.

Løsning:

$$B = \mu n I = \mu \frac{N}{l} I \quad (\mu = \mu_r \mu_0)$$

$$\text{Total omsluttet fluks: } \Phi = NBA = \frac{\mu N^2 A}{l} I = LI$$

$$\Rightarrow \underline{L = \mu N^2 A / l} = \mu \cdot 10^6 \cdot 10^{-3} \text{ m}^2 / 0,25 \text{ m} = \mu \cdot 4000 \text{ m}$$

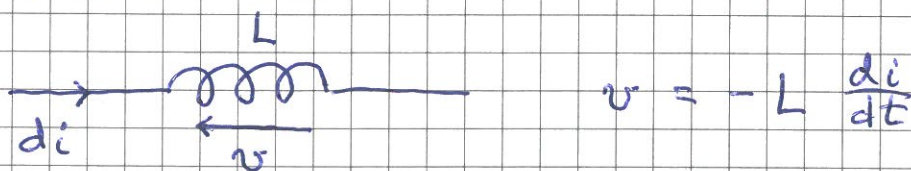
$$\text{Luftfylt: } \mu = \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m} \Rightarrow L = 4\pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} \cdot 4000 \text{ m} \\ = \underline{\underline{5,0 \text{ mH}}}$$

$$\text{Jernkjerne, } \mu_r = 2000: \mu = 8\pi \cdot 10^{-4} \text{ H/m}$$

$$\Rightarrow L = 8\pi \cdot 4 \cdot 10^{-1} \text{ H} = \underline{\underline{10 \text{ H}}}$$

Energi i \vec{B} -felt [YF 30.3; LHL 25.3]

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Påkrevd energi for å øke strøm fra i til $i + di$:

$$dU = P \cdot dt = -v \cdot i \, dt = L \frac{di}{dt} \cdot i \, dt = L i \, di$$

↑ arbeid mot
indusert v

⇒ Å øke strøm fra $i = 0$ til $i = I$ krever energien

$$U = \int dU = \int_0^I L i \, di = \underline{\underline{\frac{1}{2} L I^2}}$$

Med lang spole, lengde l , tverrsnitt A , N viklinger:

$$B = \mu_0 n I = \mu_0 \frac{N}{l} I \quad \Rightarrow \underline{I = B / \mu_0 N / l}$$

$$\Phi = \underline{NAB} = NA \mu_0 \frac{N}{l} I = \underline{LI}$$

$$\Rightarrow U = \frac{1}{2} L I \cdot I = \frac{1}{2} NAB \cdot \frac{B}{\mu_0 N / l} = \frac{1}{2\mu_0} B^2 \cdot \underbrace{(Al)}_{\substack{= \text{volum} \\ \text{inni spolen,} \\ \text{der } B \neq 0}}$$

⇒ Energi pr volumenhet i magnetfelt er

$$\boxed{u_B = \frac{1}{2\mu_0} B^2} \quad (\text{gjelder generelt})$$

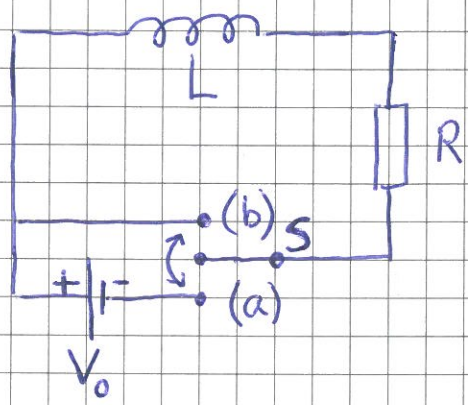
Fra før (s 112): $u_E = \frac{1}{2} \epsilon_0 E^2$

$$\Rightarrow \text{Total energitetthet i E.M. felt: } \boxed{u = u_E + u_B = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2}$$

Elektriske kredse; DC og AC; R, L og C

[YF 30.4+5+6; LHL 25.2, 27.1+2+3+5]

① RL-krets; DC

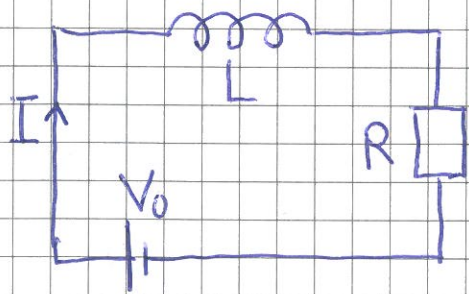


S: bryter (switch)

(a) Tilkobling af V_0

(b) Frakobling af V_0

(a) Tilkobling af V_0 ved $t=0$



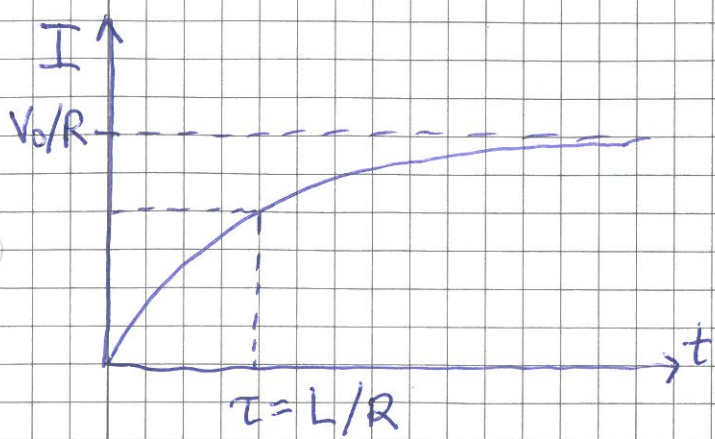
$$K2: V_0 - L \frac{dI}{dt} - RI = 0$$

Samme ligning for I som for Q i RC-krets (s 121)

$$(I(0) = 0)$$

⇒ Samme løsning!

$$\Rightarrow I(t) = \frac{V_0}{R} (1 - e^{-Rt/L})$$



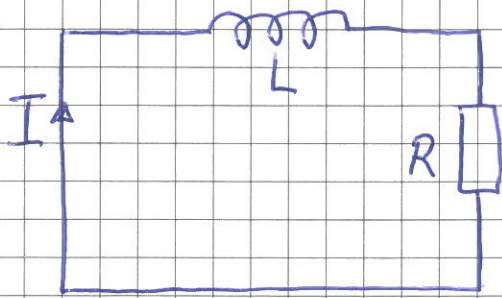
RL-kretsens tidskonstant:

$$\tau = L/R$$

(For RC-krets: $\tau = RC$)

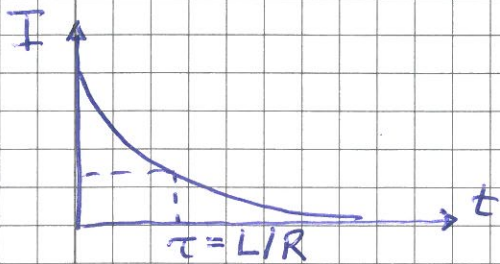
(b) Frakobling av V_0 ved "nytt $t=0$ "

$(I(0) = V_0/R)$

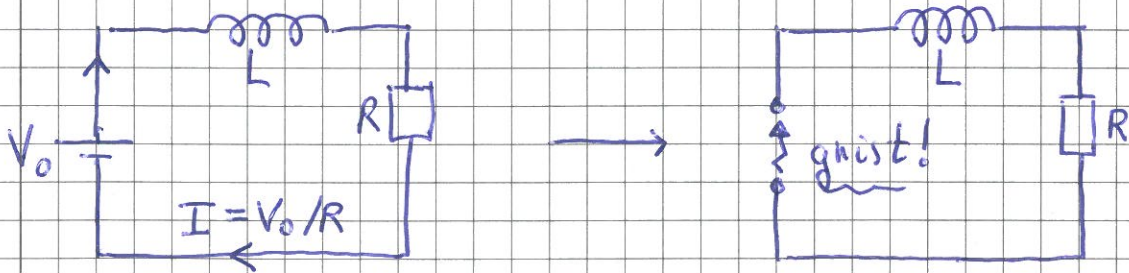


K2: $-L \frac{dI}{dt} - RI = 0$

$\Rightarrow I(t) = \frac{V_0}{R} e^{-t/\tau}; \tau = \frac{L}{R}$



(c) "Dra ut støpselet"

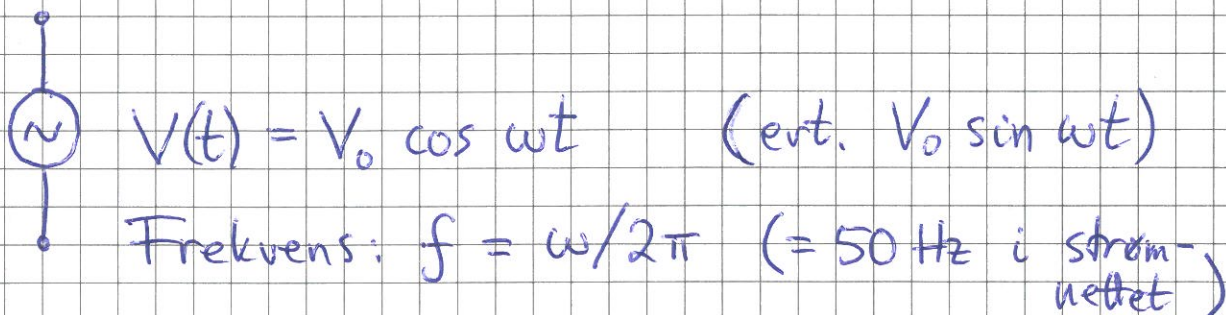


- I reduseres raskt fra V_0/R til 0 når støpselet trekkes ut av stikk-kontakten
- Stor $|dI/dt|$ gir stor induert spenning $|L dI/dt|$ (selv om L er liten)
- Gir kortvarig strøm over luftgapet (selv om R der er stor), dvs gnist (overslag).

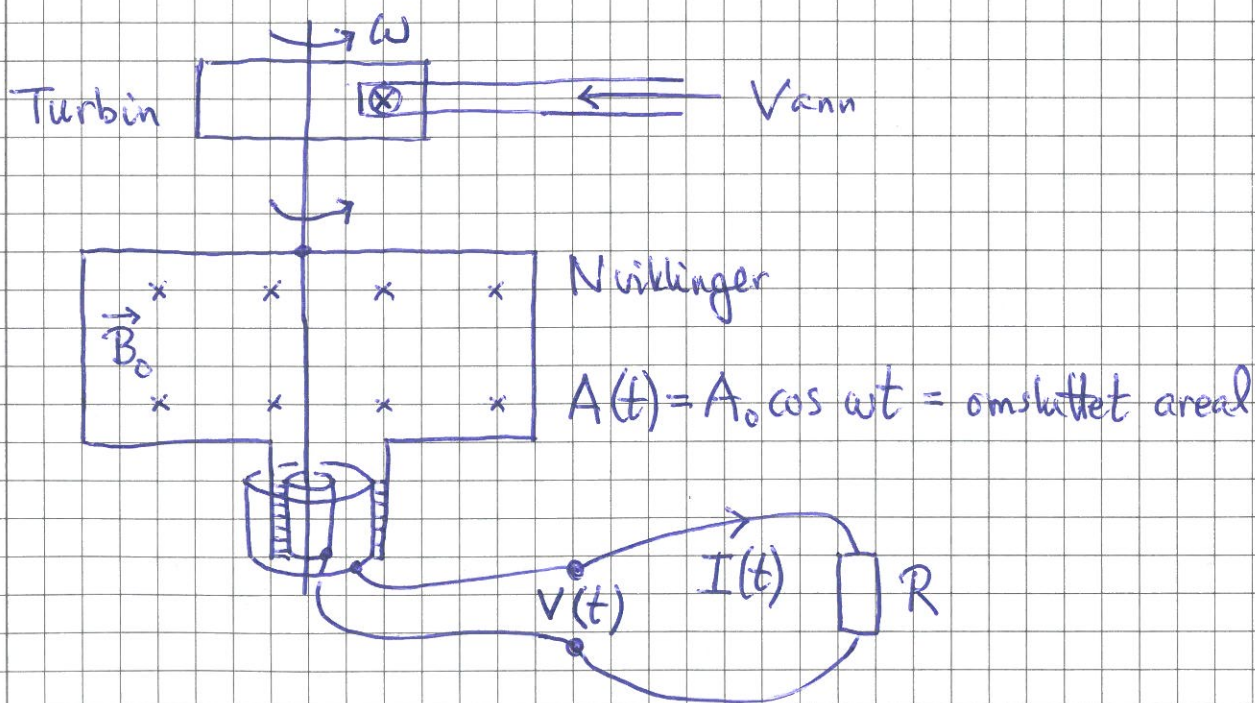
AC - kretser

- AC = alternating current = vekselstrøm

AC spenningskilde:



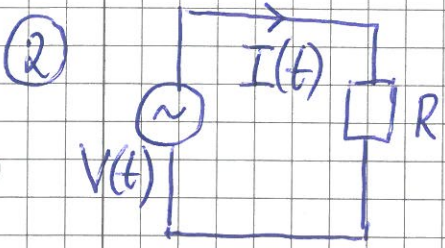
- Prinsipp for vekselstrømgenerator:



$$\Phi(t) = NB_0 A_0 \cos \omega t$$

$$V(t) = -d\Phi/dt = \underbrace{NB_0 A_0 \omega}_{V_0} \sin \omega t = V_0 \sin \omega t$$

$$I(t) = V(t)/R = \underbrace{(V_0/R)}_{I_0} \sin \omega t = I_0 \sin \omega t$$



K2: $V_0 \sin \omega t - RI(t) = 0$

$I(t) = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t$

$P(t) = V(t) \cdot I(t) = V_0 I_0 \sin^2 \omega t$

Middlere effekt:

$\langle P \rangle = \frac{1}{T} \int_0^T P(t) dt = \frac{1}{2} V_0 I_0 = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} = V_{rms} \cdot I_{rms}$

(rms: root mean square)

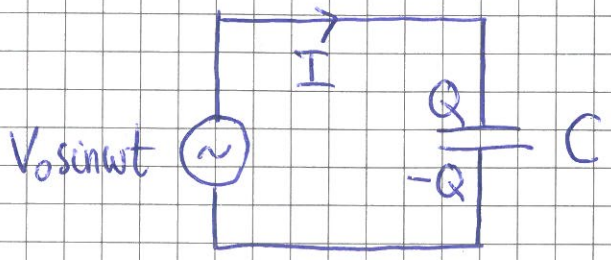
Strømnettet: $V_0 = 311 \text{ V} \Rightarrow V_{rms} = 311 \text{ V} / \sqrt{2} = 220 \text{ V}$

[Overføringsnett med gitt motstand R: $P = VI = \frac{V \cdot V}{R} = \frac{V^2}{R}$]

Energiforbruk på tid t: $W = Pt = RI^2 t$

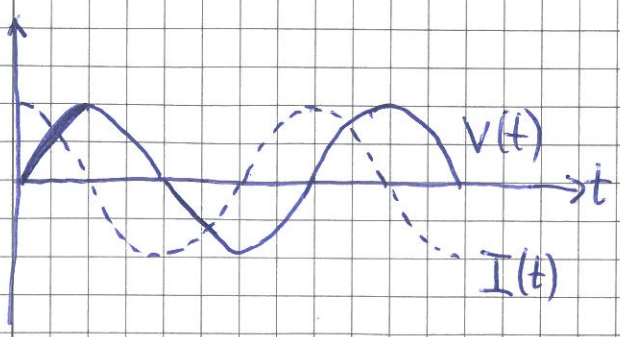
⇒ Fordel med lav strøm I og høy spenning V! Norge: 10-400 kV]

3



K2: $V_0 \sin \omega t - Q(t)/C = 0 \Rightarrow Q(t) = V_0 C \sin \omega t$

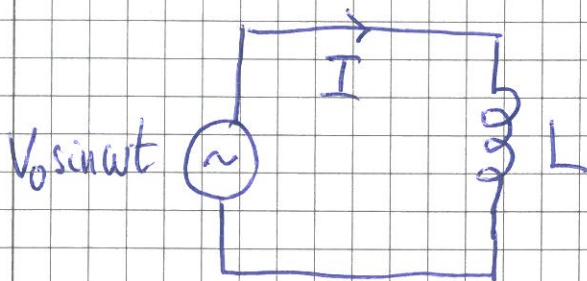
$\Rightarrow I(t) = V_0 \omega C \cos \omega t = V_0 \omega C \sin(\omega t + \frac{\pi}{2})$



• Faseforskjell $\pi/2$ mellom $V(t)$ og $I(t)$

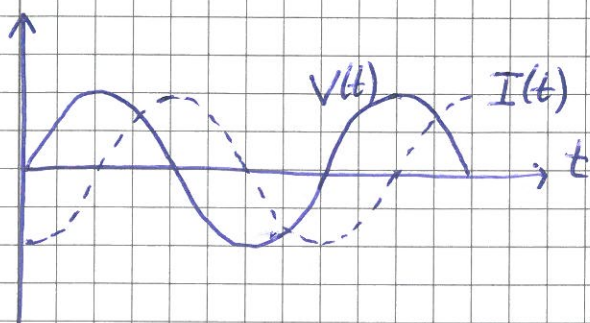
• Strøamplituden $I_0(\omega) = V_0 \omega C$ øker med frekvensen!

4



$$K2: V_0 \sin \omega t - L \frac{dI}{dt} = 0 \Rightarrow \frac{dI}{dt} = \frac{V_0}{L} \sin \omega t$$

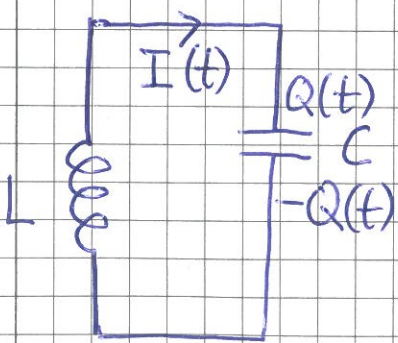
$$\Rightarrow I(t) = -\frac{V_0}{\omega L} \cos \omega t = \frac{V_0}{\omega L} \sin(\omega t - \pi/2)$$



- Faseforskytning $-\pi/2$ mellom $V(t)$ og $I(t)$

- Strømanpl. $I_0(\omega) = V_0/\omega L$ avtar med frekvensen!

5 LC-krets

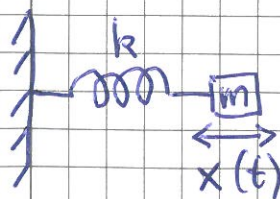


Anta $Q(0) = Q_0$

$$K2: -L \ddot{I} - \frac{Q}{C} = 0; I = \dot{Q}$$

$$\Rightarrow \underline{\underline{\ddot{Q} + \frac{1}{LC} Q = 0}}$$

Samme ligning som for $x(t)$ med harmonisk oscillator:



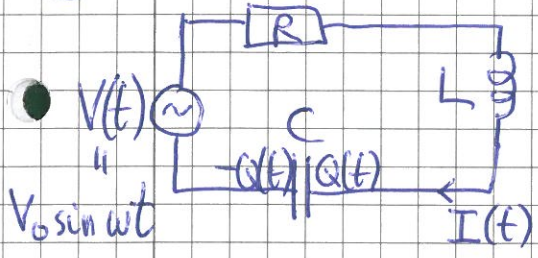
$$\ddot{x} + \frac{k}{m} x = 0$$

$$x(t) = x_0 \cos \omega_0 t; \omega_0 = \sqrt{k/m}$$

$$\Rightarrow Q(t) = Q_0 \cos \omega_0 t, \text{ med } \omega_0 = \sqrt{1/LC}$$

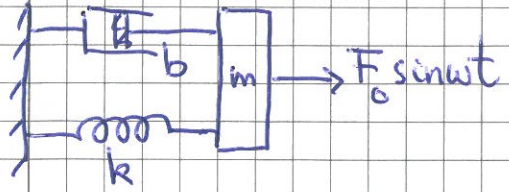
"Analoge" størrelser: $Q \leftrightarrow x; I \leftrightarrow \dot{x}; L \leftrightarrow m; \frac{1}{C} \leftrightarrow k$

⑥ RLC resonanskrets



K2: $V_0 \sin \omega t - RI - LI - Q/C = 0$
 $\Rightarrow L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = V_0 \sin \omega t$

Mekanisk analogi:



N2 $\Rightarrow m\ddot{x} + b\dot{x} + kx = F_0 \sin \omega t$
 (dvs $b \leftrightarrow R$; $F_0 \leftrightarrow V_0$)

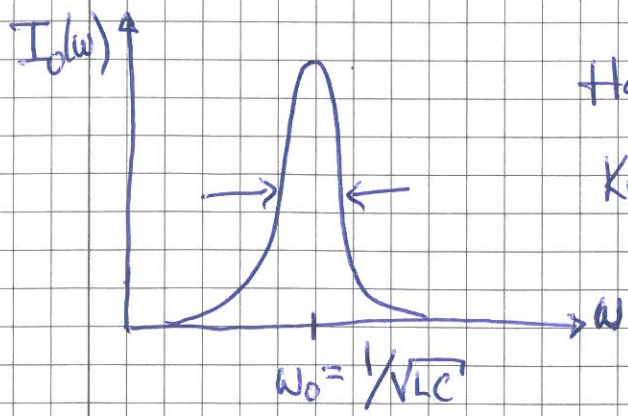
For mek. system: $x(t) = A(\omega) \sin(\omega t + \varphi)$

med $A(\omega) = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\gamma\omega)^2}}$; $\omega_0 = \sqrt{k/m}$, $2\gamma = b/m$

Dermed, for RLC-kretsen: $Q(t) = Q_0(\omega) \sin(\omega t + \varphi)$

med $Q_0(\omega) = \frac{V_0/L}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\gamma\omega)^2}}$; $\omega_0 = \sqrt{1/LC}$, $2\gamma = R/L$

$\Rightarrow I_0(\omega) = \omega Q_0(\omega)$



Halvverdi-bredde: $\Delta\omega \approx 2\gamma = R/L$

Kvalitetsfaktor: $\frac{\omega_0}{\Delta\omega} = \sqrt{LC}/R$

[Kan måle $I_0(\omega)$ via V_R over R :

$I(t) = V_R(t)/R = V_{R0}(\omega) \cos(\omega t + \varphi) \Rightarrow I_0(\omega) = \frac{V_{R0}(\omega)}{R}$]