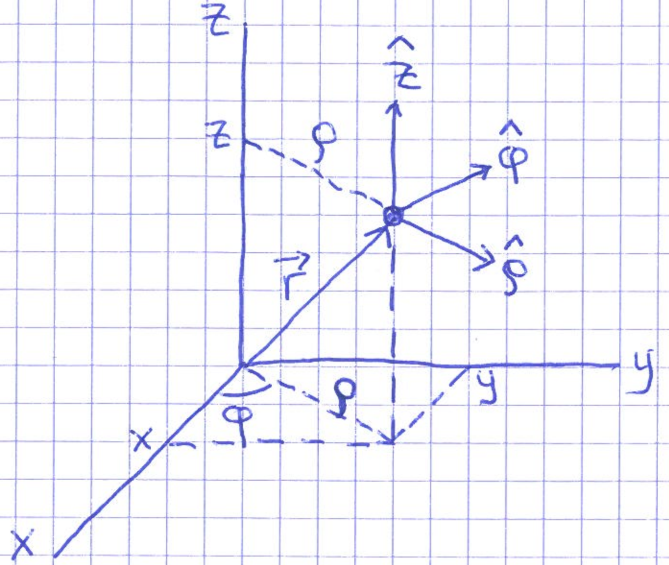
Sirkelbevegelse [YF 9.1–9.3; LL 1.8]

Antar rotasjon om gitt akse, z-aksen, og bruker sylinderkoord. (= polarkoord. + z)



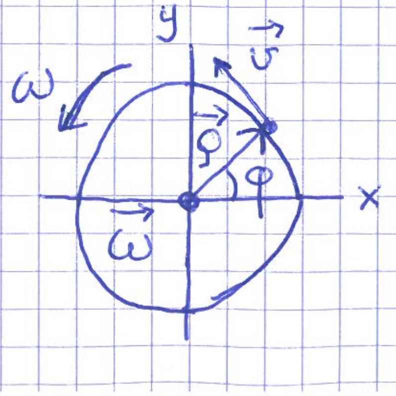
$$\vec{r} = \rho\hat{\rho} + z\hat{z} \quad (= x\hat{x} + y\hat{y} + z\hat{z})$$

$$x = \rho\cos\varphi, \quad y = \rho\sin\varphi, \quad z = z$$

$$\rho = \sqrt{x^2+y^2}, \quad \tan\varphi = y/x$$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{\rho^2+z^2}$$

Lar vinkelhastigheten bli vektor som peker langs rotasjonsaksen:

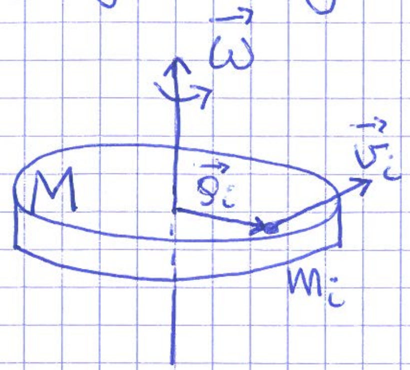


$$\vec{\omega} = \omega\hat{z} \quad (= \omega\hat{\omega}), \quad \vec{\rho} = \rho\hat{\rho}$$

$$\vec{v} = \rho\omega\hat{\varphi}$$

Ser at $\boxed{\vec{v} = \vec{\omega} \times \vec{\rho}}$ gir riktig retning ($\hat{\varphi} = \hat{z} \times \hat{\rho}$) og tallverdi ($v = \omega\rho$)

Rotasjonsenergi [YF 9.4; LL 6.4]



$$K = K_{rot} = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2$$
$$= \sum_i \frac{1}{2} m_i (\rho_i \omega)^2 = \underline{\frac{1}{2} \left\{ \sum_i m_i \rho_i^2 \right\} \omega^2}$$

Treghetsmoment [YF 9.4; LL 6.3]

$I = \sum_i m_i \rho_i^2$ = legemets treghetsmoment mhp gitt akse

Med kont. massefordeling:

$m_i \rightarrow \Delta m_i \xrightarrow{\Delta m_i \to 0} dm$ og $\sum_i \longrightarrow \int$

$$\Rightarrow \quad \boxed{I = \int \rho^2 \, dm}$$

(der ρ = avstand fra aksen til dm)

Dermed:

$$\boxed{K_{rot} = \frac{1}{2} I \omega^2}$$

Kinetisk energi for stivt legeme [YF 10.3; LL 6.6]

Generell bevegelse: Translasjon av CM + Rotasjon om CM

$$\Rightarrow \boxed{K = K_{trans} + K_{rot} = \frac{1}{2} M V^2 + \frac{1}{2} I_0 \omega^2}$$

M = massen, $V = \dot{\vec{R}}_{CM}$,

I_0 = treghetsmoment mhp rot.aksen gjennom CM

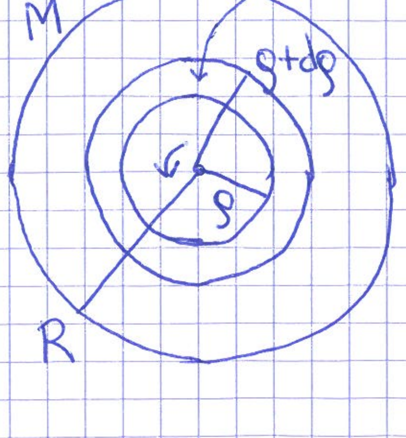
$\vec{\omega}$ = vinkelhast. for rot. omkring —— " ——

Eksempler på beregning av I [YF 9.6; LL 6.3]

Eks 1: Ring (evt. sylinderskall)

$$I_0 = \int_{ring} \rho^2 \, dm = R^2 \int_{ring} dm = \underline{\underline{MR^2}}$$

Eks 2: Sirkulær skive (evt. kompakt sylinder)

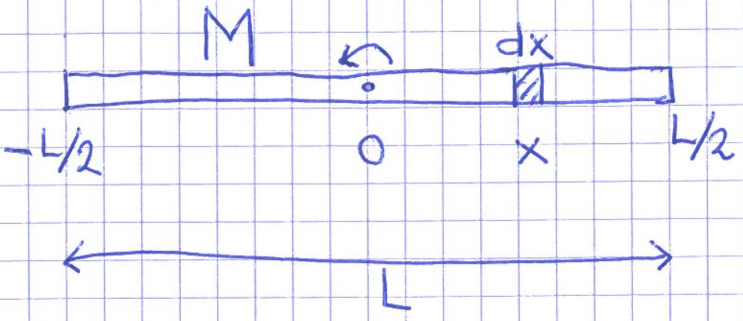


$$dI_0 = dm \cdot \rho^2 \; ; \quad dm = M \frac{dA}{A} = M \cdot \frac{2\pi \rho \, d\rho}{\pi R^2}$$

$$\Rightarrow I_0 = \int dI_0 = \int_0^R M \frac{2\pi \rho \, d\rho}{\pi R^2} \rho^2$$

$$= \frac{2M}{R^2} \underbrace{\int_0^R \rho^3 \, d\rho}_{R^4/4} = \underline{\underline{\frac{1}{2} M R^2}}$$

Eks 3: Tynn stang



$$\rho = x, \quad dm = M\,dx/L$$

$$\Rightarrow I_0 = \int_{-L/2}^{L/2} x^2 M\,dx/L = \frac{M}{L}\Big|_{-L/2}^{L/2} \frac{x^3}{3} = \underline{\underline{\frac{1}{12}ML^2}}$$

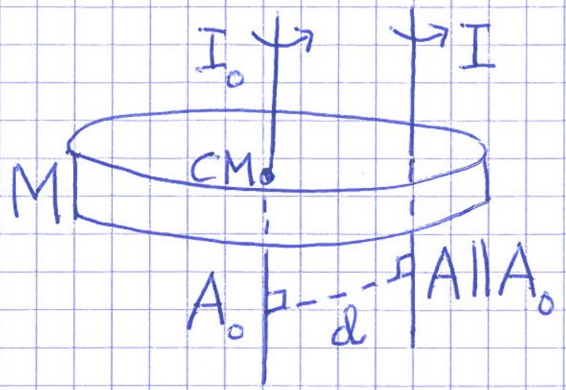
Eks 4: Mhp akse gjennom stangas ende

$$I = \int_0^L x^2 M\,dx/L = \underline{\underline{\frac{1}{3}ML^2}}$$

Eks 5: Kuleskall: $I_0 = \frac{2}{3}MR^2$

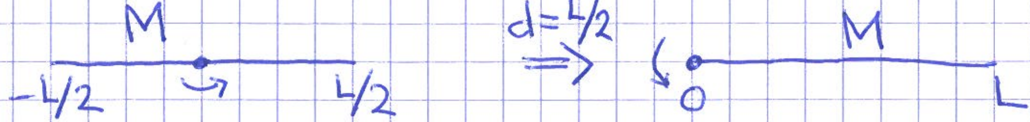
Eks 6: Kompakt kule: $I_0 = \frac{2}{5}MR^2$

Steiners sats [YF 9.5; LL 6.3]



$$\boxed{I = I_0 + Md^2}$$

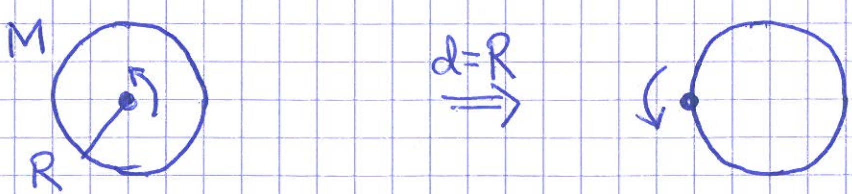
Eks 1: Tynn stang



$I_0 = \frac{1}{12}ML^2$

$I = I_0 + M(L/2)^2 = \frac{1}{3}ML^2$

Eks 2: Kuleskall

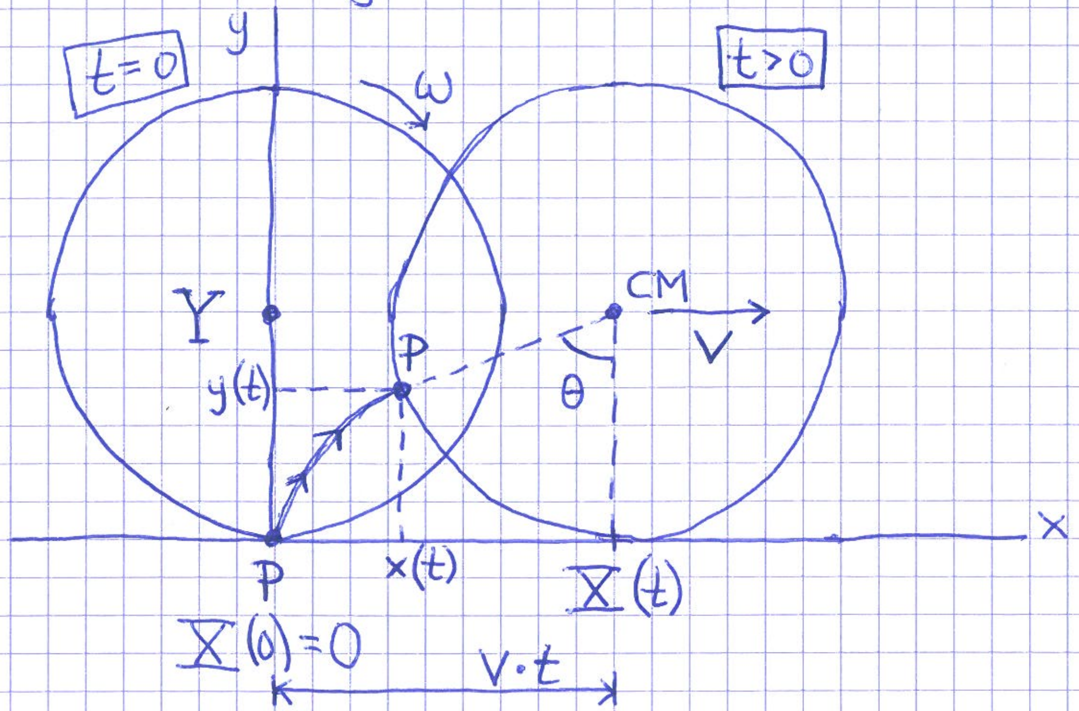


$I_0 = \frac{2}{3}MR^2$

$I = I_0 + MR^2 = \frac{5}{3}MR^2$

Rulling og sluring [YF 10.3 ; LL 6.7]

Ren rulling



P = punkt på periferien ; $(x(t), y(t))$ = banen til P

$$V = \dot{R}_{CM} = \dot{X} \quad ; \quad Y = R$$

$$\omega = \dot{\theta}$$

$$X = V \cdot t = R \cdot \theta$$

$$V = \dot{X} = R\dot{\theta} = R\omega$$

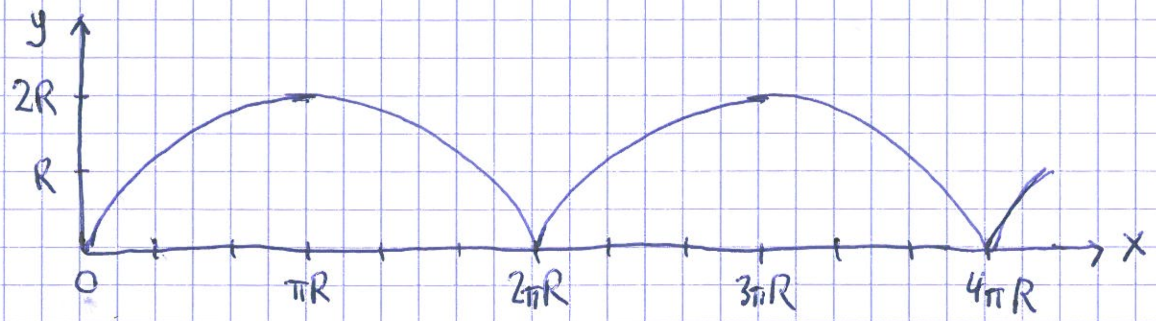
$$A = \ddot{X} = \dot{V} = R\ddot{\theta} = R\dot{\omega} = R\alpha$$

Rullebetingelser : $\boxed{V = R\omega \, , \quad A = R\alpha}$

Banen til P:

$$x = X - R\sin\theta \quad , \quad y = R - R\cos\theta$$

Sykloide:

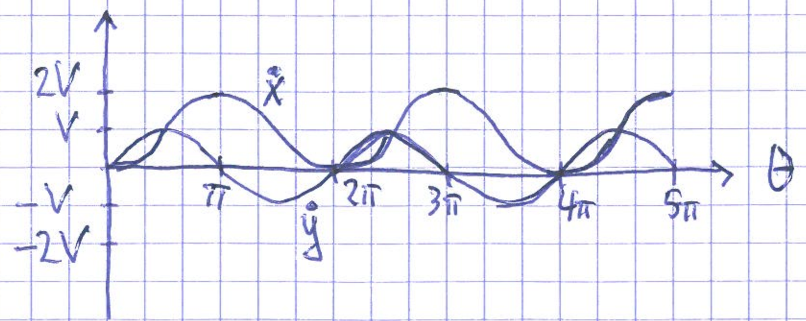


Hastigheten til P:

$$\dot{x} = \dot{X} - R\dot{\theta}\cos\theta = V(1-\cos\theta)$$

$$\dot{y} = R\dot{\theta}\sin\theta = V\sin\theta$$

$$\Rightarrow \vec{v}(\theta) = \hat{x}\,V(1-\cos\theta) + \hat{y}\,V\sin\theta$$



K ved ren rulling:

$$I_0 = c\cdot MR^2 \quad (c=1 \text{ for ring etc}) \quad ; \quad \omega = V/R$$

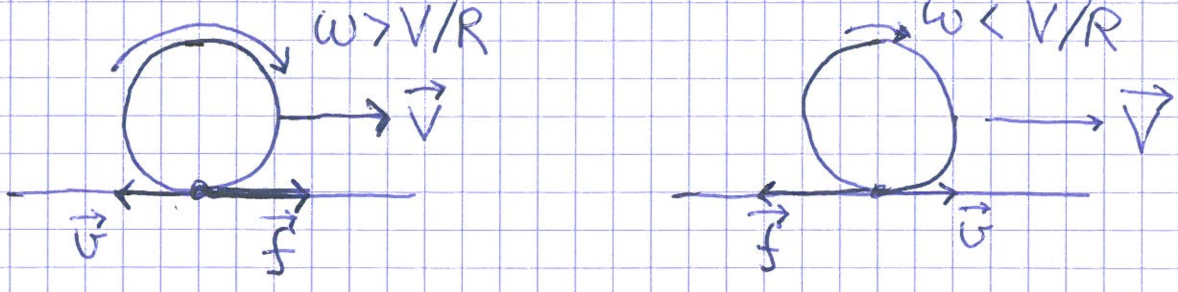
$$\Rightarrow K = \frac{1}{2}MV^2 + \frac{1}{2}cMR^2\frac{V^2}{R^2} = \underline{\underline{(1+c)\cdot\frac{1}{2}MV^2}}$$

Sluring

$\omega \neq V/R \Rightarrow$ relativ hastighet $v = V - \omega R \neq 0$

mellom legeme og underlag i kontaktpunktet

$\Rightarrow$ legemet roterer og glir ; legemet slurer



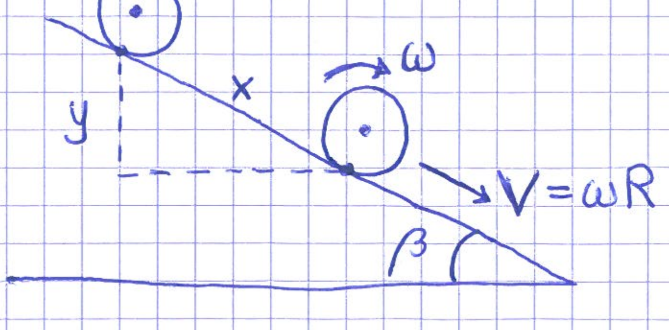
Effekttap: $P_f = \vec{f} \cdot \vec{v} < 0 \qquad (f = |\vec{f}| = \mu_k N)$

Hvis ren rulling, $v = 0$: $P_f = \vec{f} \cdot \vec{v} = 0$

Statisk friksjon $f \leq \mu_s N$; $\vec{f}$ rettet mot "tenkt relativhast." $\vec{v}$ hvis det ikke hadde vært friksjon.

[I praksis litt tap av mek. energi også ved ren rulling pga "rullefriksjon".]

Eks: Ren rulling på skråplan [YF 10.3 ; LL 6.8]



M = masse, R = radius, $I_0 = cMR^2$

Finn $\dot{V}$, samt minste tillatte μ_s for ren rulling.

Exp viser: $\dot{V}(\text{kule}) > \dot{V}(\text{skive}) > \dot{V}(\text{kuleskall}) > \dot{V}\left(\substack{\text{hul} \\ \text{sylinder}}\right)$

Løsning: E er bevart $\Rightarrow Mgy = (1+c)\cdot\frac{1}{2}MV^2$

$$y = x\sin\beta$$

$$\Rightarrow V = \sqrt{2gx\sin\beta/(1+c)}$$

Objekt	kule	skive	kuleskall	hul sylinder
c	2/5	1/2	2/3	1

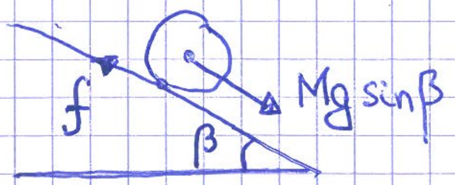
$\Rightarrow$ exp. er forklart!

Akselerasjonen:

$$\dot{V} = \sqrt{\frac{2g\sin\beta}{1+c}}\cdot\frac{d}{dt}\sqrt{x} \quad ; \quad \frac{d}{dt}\sqrt{x} = \frac{1}{2}x^{-1/2}\cdot\frac{dx}{dt} = \frac{V}{2\sqrt{x}}$$

$$\Rightarrow \dot{V} = \sqrt{\frac{2g\sin\beta}{1+c}}\cdot\frac{\sqrt{2gx\sin\beta/(1+c)}}{2\sqrt{x}} = \underline{\underline{g\cdot\frac{\sin\beta}{1+c}}}$$

Hvis bare $G_{\parallel} = Mg\sin\beta$ virket langs skråplanet, ville $\dot{V}$ ha blitt $g\sin\beta$. Friksjon f, rettet oppover skråplanet, reduserer $\dot{V}$ med faktoren $1/(1+c)$:



$$Mg\sin\beta - f = M\dot{V} = \frac{Mg\sin\beta}{1+c}$$

$$\Rightarrow f = Mg\sin\beta\cdot\left\{1 - \frac{1}{1+c}\right\}$$

$$= \frac{c}{1+c}Mg\sin\beta$$

Men: $f_{max} = \mu_s N = \mu_s Mg\cos\beta$

$\Rightarrow$ Ren rulling bare mulig hvis

$$\mu_s Mg\cos\beta \geq \frac{c}{1+c}Mg\sin\beta$$

$$\underline{\underline{\mu_s \geq \frac{c}{1+c}\tan\beta}}$$

Kule (kompakt): $\mu_s \geq \frac{2}{7}\tan\beta$ osv.