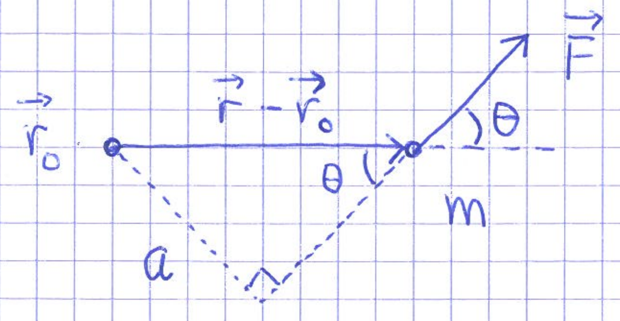
Rotasjonsdynamikk

Dreiemoment [YF 10.1 ; LL 5.5, 6.4]



$$\boxed{\vec{\tau} = (\vec{r} - \vec{r}_0) \times \vec{F}}$$ = $\vec{F}$'s dreiemoment på m, relativt valgt referansepunkt $\vec{r}_0$

Retning:

$\vec{\tau} \perp \vec{F}$ og $\vec{\tau} \perp \vec{r} - \vec{r}_0$ (ut av planet i figuren)

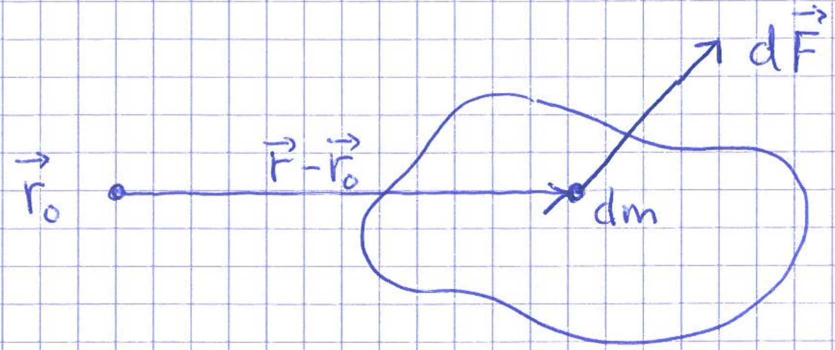
Absoluttverdi:

$$|\vec{\tau}| = |\vec{r} - \vec{r}_0| \cdot |\vec{F}| \cdot \sin\theta$$

$$= a \cdot F$$

$$(= \text{arm} \cdot \text{kraft})$$

For partikkelsystem:



$$d\vec{\tau} = (\vec{r}-\vec{r}_0) \times d\vec{F}$$

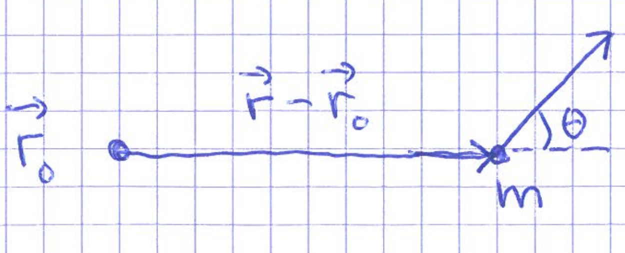
$$\vec{\tau} = \int d\vec{\tau} = \int (\vec{r}-\vec{r}_0) \times d\vec{F}$$

= totalt dreiemoment på legemet, relativt $\vec{r}_0$

Fra før: Netto <u>ytre $\vec{F}$</u> $\Rightarrow$ endring i <u>impulsen $\vec{p}$</u>

Nå: Netto <u>ytre $\vec{\tau}$</u> $\Rightarrow$ endring i <u>dreieimpulsen $\vec{L}$</u>

<u>Dreieimpuls</u> [YF 10.5 ; LL 6.6]



$$\vec{p} = m\vec{v}$$

$$\boxed{\vec{L} = (\vec{r}-\vec{r}_0) \times \vec{p}}$$ = m's dreieimpuls, relativt $\vec{r}_0$

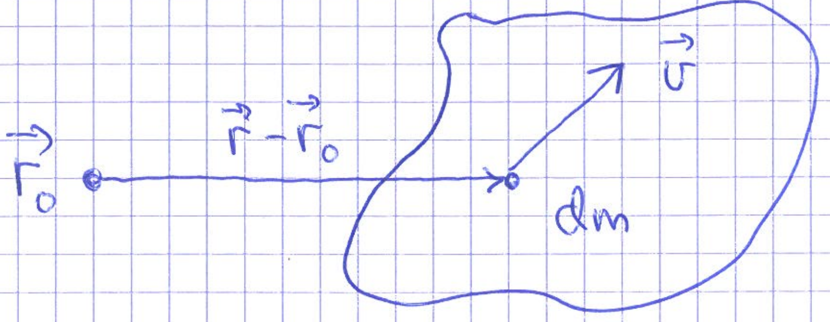
Retning:

$$\vec{L} \perp \vec{p} \quad \text{og} \quad \vec{L} \perp \vec{r}-\vec{r}_0$$

Absolüttverdi:

$$|\vec{L}| = |\vec{r}-\vec{r}_0| \cdot |\vec{p}| \cdot \sin\theta = a \cdot p \quad (\text{arm} \cdot \text{impuls})$$

For partikkelsystem:



$$d\vec{L} = (\vec{r}-\vec{r}_0) \times d\vec{p} = dm\,(\vec{r}-\vec{r}_0) \times \vec{v}$$

$$\vec{L} = \int d\vec{L} = \int dm\,(\vec{r}-\vec{r}_0) \times \vec{v}$$

= legemets totale dreieimpuls, relativt $\vec{r}_0$

N2 for rotasjon [YF 10.5 ; LL 6.6]

N2 : $\vec{F} = m\vec{a} = m\dot{\vec{v}} = \dot{\vec{p}}$

Anta fast $\vec{r}_0$, evt. $\dot{\vec{r}}_0 \parallel \vec{v}$, for da er $\dot{\vec{r}}_0 \times \vec{v} = 0$

$$\dot{\vec{L}} = \frac{d}{dt}\left\{ m(\vec{r}-\vec{r}_0)\times\vec{v} \right\} = \underbrace{m\vec{v}\times\vec{v}}_{=0} + m(\vec{r}-\vec{r}_0)\times\vec{a}$$

$$\overset{N2}{=} (\vec{r}-\vec{r}_0)\times\vec{F} \overset{(def)}{=} \vec{\tau}$$

For partikkelsystem :

$$\dot{\vec{L}} = \frac{d}{dt}\int dm\,(\vec{r}-\vec{r}_0)\times\vec{v} = \int dm\,(\vec{r}-\vec{r}_0)\times\vec{a}$$

$$\overset{N2}{=} \int (\vec{r}-\vec{r}_0)\times d\vec{F} = \int d\vec{\tau} = \vec{\tau}$$

$$\Rightarrow \quad \boxed{\vec{\tau} = d\vec{L}/dt} \quad \text{N2, rotasjon}$$

der

$\vec{\tau}$ = netto ytre dreiemoment på legemet

$\vec{L}$ = legemets dreieimpuls

NB: <u>Samme</u> ref.punkt $\vec{r}_0$ i $\vec{\tau}$ og $\vec{L}$!

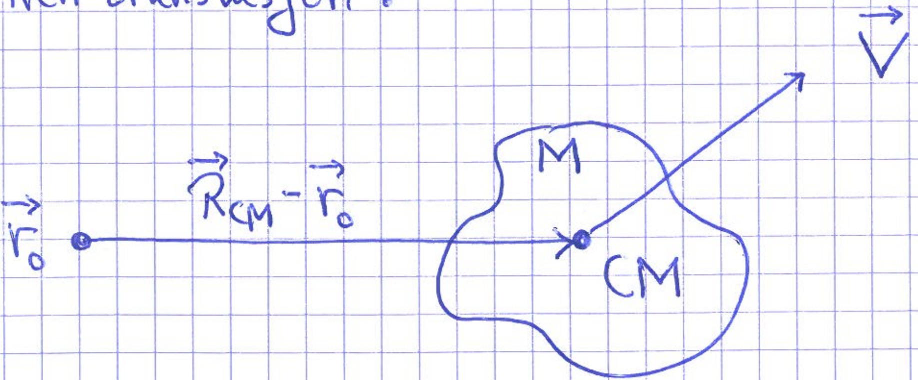
Bevaringslover, oppsummert

- $\vec{F}_{ytre} = 0$ (isolert system) $\Rightarrow$ $E_{(total)}$, $\vec{p}$ og $\vec{L}$ bevart
  (ingen ytre krefter) $\leftrightarrow$

- Konservativt system $\Rightarrow$ mek. energi $E = K + U$ bevart
- $\sum \vec{F}_{ytre} = 0 \Rightarrow \vec{p}$ bevart
- $\sum \vec{\tau}_{ytre} = 0 \Rightarrow \vec{L}$ bevart

$\vec{L}$ for stivt legeme [YF 10.5 ; LL 6.6]

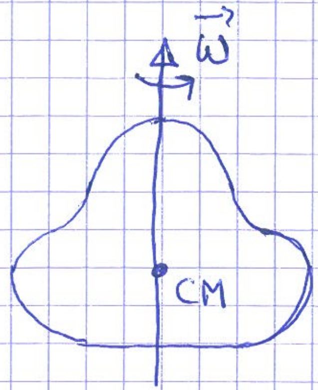
Ren translasjon:



$$\xRightarrow[\text{def.}]{\text{Fra}} \quad \vec{L}_b = M\left(\vec{R}_{CM} - \vec{r}_0\right) \times \vec{V}$$

$$= \underline{\text{b}}\text{anedreieimpuls}$$

Ren rotasjon; om akse gjennom CM:



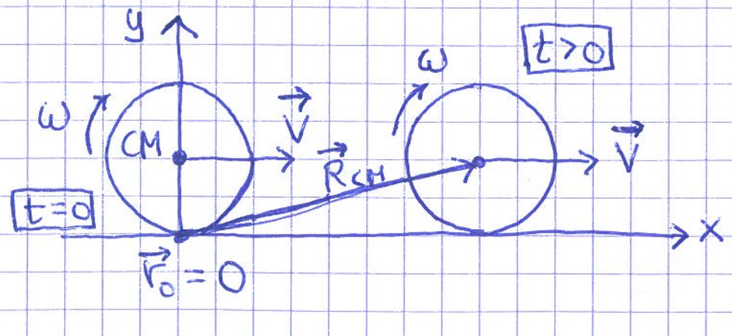
Med refleksjonssymmetri om rotasjonsaksen er

$\vec{L}_s = I_0 \vec{\omega}$ = indre dreieimpuls, spinn; uavhengig av $\vec{r}_0$

Total dreieimpuls for stivt legeme:

$$\boxed{\vec{L} = \vec{L}_b + \vec{L}_s = M(\vec{R}_{CM} - \vec{r}_0) \times \vec{V} + I_0 \vec{\omega}}$$

Eks: Snookerkule, masse M, radius R, $I_0 = \frac{2}{5}MR^2$



$\vec{\omega} = -\omega \hat{z}$

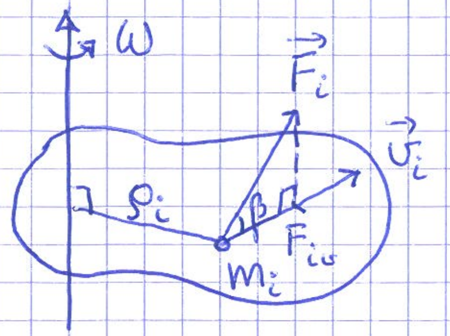
$\boxed{\vec{L} = ?}$

$$\vec{L} = M\vec{R}_{CM} \times \vec{V} + I_0 \vec{\omega}$$

$$= -MRV\hat{z} - \frac{2}{5}MR^2\omega\hat{z}$$

Hvis ren rulling: $V = \omega R \Rightarrow \vec{L} = -\frac{7}{5}MR^2\omega\hat{z}$

N2 for rotasjon om akse med fast orientering



$$\vec{F}_i \cdot \vec{v}_i = F_i v_i \cos\beta$$
$$= F_{iv} \cdot v_i$$

Skal se at $\underline{\tau = I\dot{\omega}}$ ved å regne ut to forskjellige uttrykk for effekten som tilføres det stive legemet.

$$P = \sum_i \vec{F}_i \cdot \vec{v}_i \overset{N2}{=} \sum_i m_i \frac{d\vec{v}_i}{dt} \cdot \vec{v}_i \overset{s.21}{=} \frac{d}{dt} \sum_i \frac{1}{2} m_i v_i^2 \left(= \frac{dK}{dt}\right)$$

$$= \frac{d}{dt} \left\{ \sum_i m_i \rho_i^2 \right\} \cdot \frac{1}{2}\omega^2 = I \cdot \frac{1}{2} \cdot 2\omega \cdot \frac{d\omega}{dt}$$

$$P = \sum_i \vec{F}_i \cdot \vec{v}_i = \sum_i F_{iv} v_i = \left\{ \sum_i F_{iv} \rho_i \right\} \omega = \tau\omega$$

$$\Rightarrow \boxed{\tau = I\dot{\omega}}$$ (N2, rot. om akse m/fast orientering)

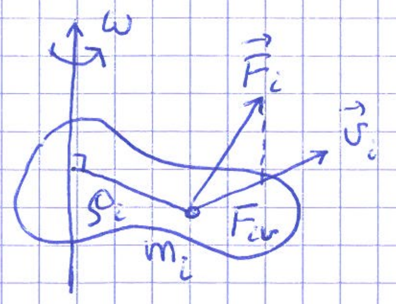
der $\tau = \sum_i F_{iv} \rho_i$ = ytre dreiemoment på legemet,

(her referert til rotasjons<u>aksen</u>, og ikke et ref. <u>punkt</u> $\vec{r}_0$)

$I = \sum_i m_i \rho_i^2$ = legemets treghetsmoment

mhp den ~~vilk~~ fast orienterte rot. aksen

Arbeid utført ved rotasjon [YF 10.4; LL 6.4]



Fra sist: $P = \sum_i \vec{F}_i \cdot \vec{v}_i = \tau\omega$

med $\tau = \sum_i F_{iv}\, \rho_i$

$P = dW/dt$ og $\omega = d\varphi/dt$

$\Rightarrow$ Tilført effekt: $\boxed{P = \tau\omega}$

Utført arbeid ved rotasjon $d\varphi$: $\boxed{dW = \tau\, d\varphi}$

[Sammenlign translasjon: $P = \vec{F}\cdot\vec{v}$, $dW = \vec{F}\cdot d\vec{r}$]

Statisk likevekt [YF 11.1–11.3; LL 7.1]

Et stivt legeme er (og forblir!) i ro, dvs

$$\vec{p} = 0 \quad \text{og} \quad \vec{L} = 0,$$

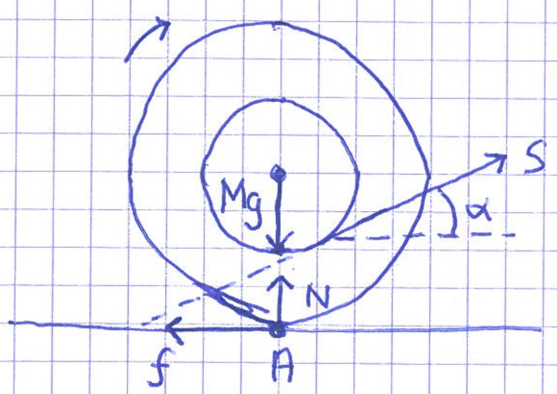
bare dersom

$$\underbrace{\sum_i \vec{F}_i}_{\text{Netto ytre kraft}} = 0 \quad \text{og} \quad \underbrace{\sum_i \vec{\tau}_i}_{\text{Netto ytre dreiemoment}} = 0$$

Rotasjon, eksempler

Eks 1: Snelle

Løsn: Kun S har arm mhp kontaktpunktet A:



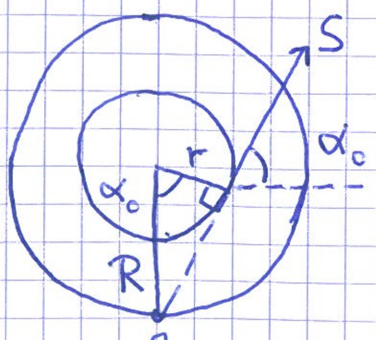
Liten α $\Rightarrow$ $\vec{\tau}_A$ inn i planet

$\Rightarrow$ Rotasjon med klokka

$\Rightarrow$ Ruller mot høyre

Stor α $\Rightarrow$ Omvendt

Statisk likevekt hvis forlengelse av $\vec{S}$ gjennom A:

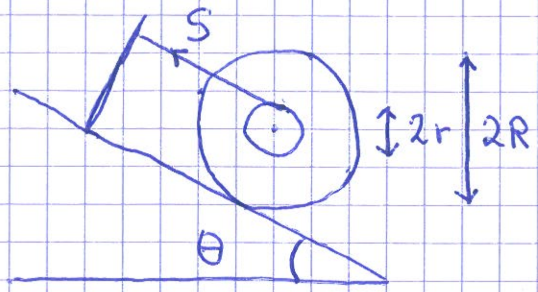


$$\cos\alpha_0 = r/R$$

Snelle i ro inntil $S \geq \dfrac{\mu_s Mg}{\frac{r}{R} + \mu_s\sqrt{1 - r^2/R^2}}$

(Vis dette)

Eks 2: Snelle på skråplanet



Finn vinkel θ_0 når snella begynner å gli.

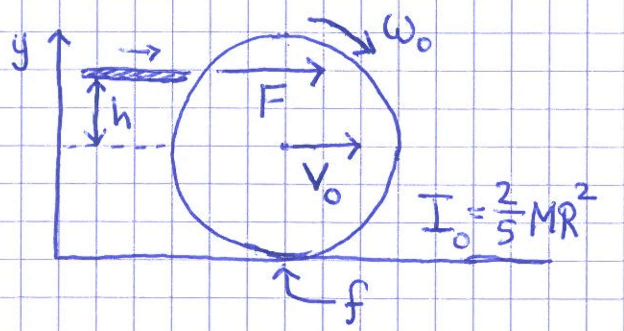
Hva er S når den glir?

Hva er a når den glir?

Strategi: N1 langs skråplanet og N1, rot. om CM, $f = f_{max}$ $\Rightarrow \theta_0$

N2 —— " —— og N2 —— " ——, $f = \mu_k N$ $\Rightarrow a$

Eks 3: Snooker [LL 6.7]



Kort støt:

N2: $F\Delta t = \Delta p = MV_0$

N2, rot om CM: $\tau \Delta t = \Delta L = I_0 \omega_0$

$\tau = Fh \quad (F \gg f)$

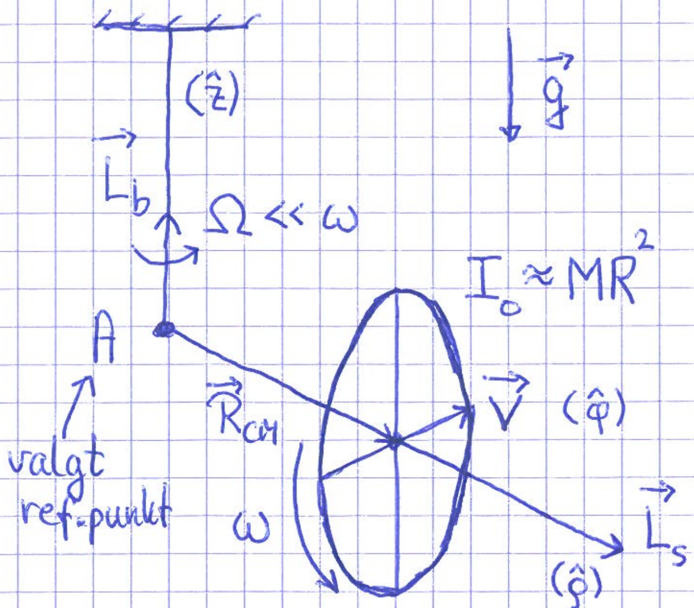
Stor h $\Rightarrow$ $\omega_0 > V_0/R$ $\Rightarrow$ skuring og $\vec{f}$ mot høyre

Liten h $\Rightarrow$ omvendt

$h = h_0$ $\Rightarrow$ $V_0 = \omega_0 R$ og ren rulling umiddelbart

Etter hvert ren rulling uansett!

Eks 4: Presesjon [YF 10.7 ; LL 6.10]



Exp:

$M \approx 5\ \text{kg}$

$R \approx 0.3\ \text{m}$

$R_{CM} \approx 0.2\ \text{m}$

$T_\Omega = \frac{2\pi}{\Omega} \approx$ ____

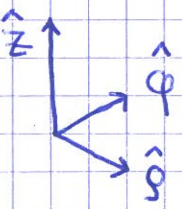
$\boxed{\omega = ?}$

Løsn: $\vec{L}_A = \vec{L}_b + \vec{L}_s = \vec{R}_{CM} \times M\vec{V} + I_0 \vec{\omega}$

$$L_b = R_{CM} MV \ll L_s = MR^2 \omega$$

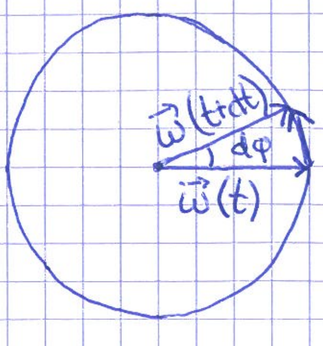
$$\vec{\tau}_A = \vec{R}_{CM} \times M\vec{g}$$

$$= R_{CM} Mg\ \hat{\varphi}$$



N2, rot. mhp A:

$$\vec{\tau}_A = \frac{d\vec{L}_A}{dt} \approx \frac{d\vec{L}_s}{dt} = I_0 \frac{d\vec{\omega}}{dt} \qquad \left(\frac{d\vec{L}_b}{dt} \approx 0\right)$$



$$d\vec{\omega} = \omega\, d\varphi\, \hat{\varphi}$$

$$\Rightarrow \frac{d\vec{\omega}}{dt} = \omega \frac{d\varphi}{dt} \hat{\varphi} = \omega \Omega \hat{\varphi}$$

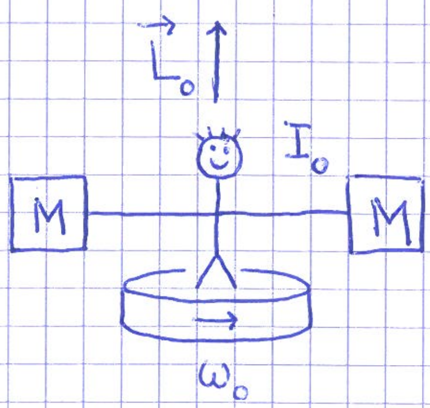
$$\Rightarrow \quad R_{CM} M g = I_0 \omega \Omega = M R^2 \omega \Omega$$

$$\Rightarrow \quad \omega = \frac{R_{CM}\, g}{R^2 \Omega}$$

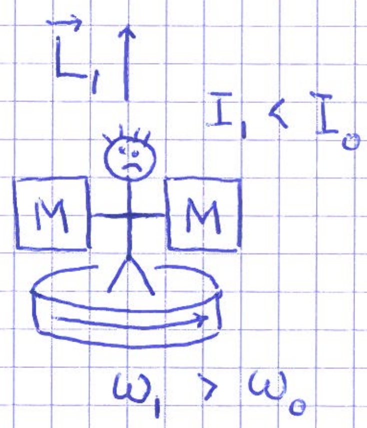
$$\Rightarrow \quad \underline{\underline{T_\omega = \frac{2\pi}{\omega} = \frac{(2\pi R)^2}{R_{CM}\, g\, T_\Omega}}}$$

Tallverdi: $T_\omega \approx \dfrac{(2\pi \cdot 0.3)^2}{0.2 \cdot 10 \cdot} \approx$ ______ s

Eks 5: Piruett [YF 10.6; LL 6.5]



$\longrightarrow$

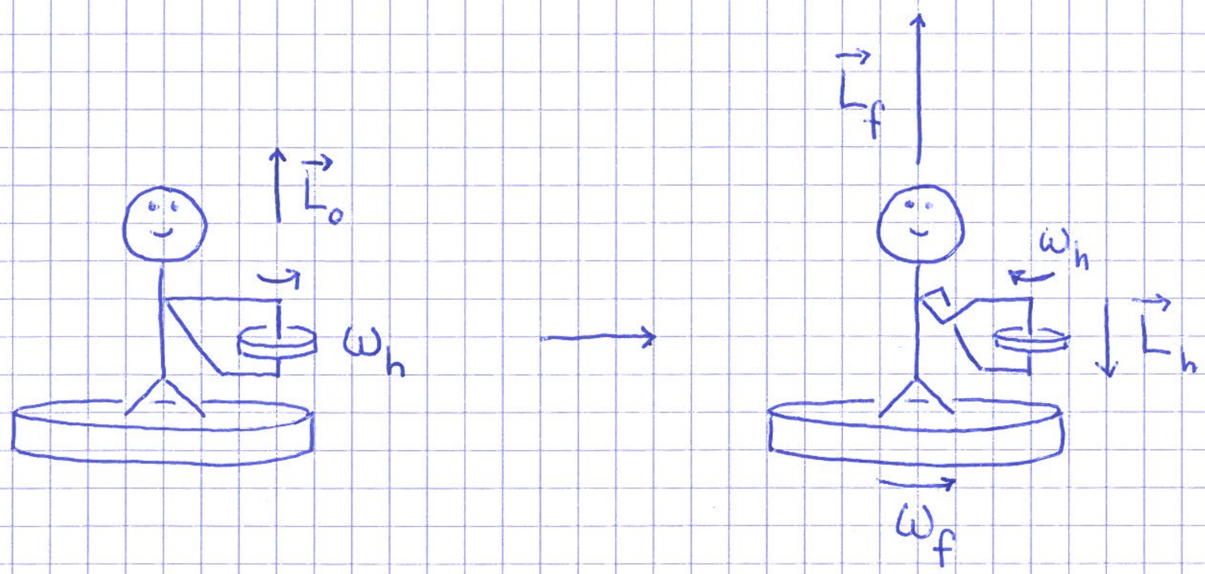


$$\vec{\tau}_{ytre} = 0 \Rightarrow \vec{L}_1 = \vec{L}_0$$

$$\Rightarrow I_1 \omega_1 = I_0 \omega_0 \quad \Rightarrow \omega_1 = \frac{I_0}{I_1}\omega_0 > \omega_0$$

[K øker! Hvorfor/hvordan?]

Eks 6: Dreieimpulsbevarelse for roterende foreleser



$$\vec{\tau}_{ytre} = 0 \quad \Rightarrow \quad \vec{L}_f + \vec{L}_h = \vec{L}_0$$

$$\vec{L}_h = -\vec{L}_0$$

$$\Rightarrow \quad \vec{L}_f = 2\vec{L}_0$$