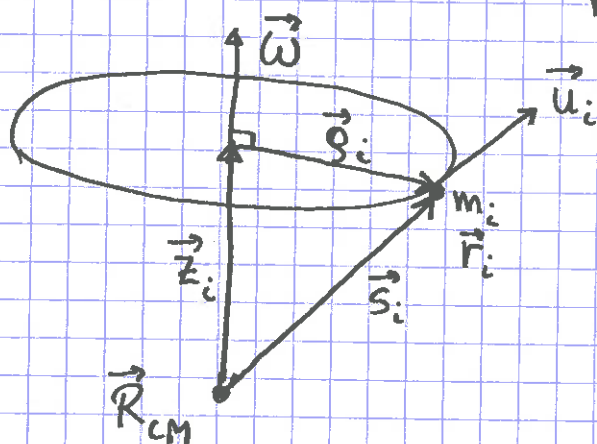


Beris for  $\vec{L} = M(\vec{R}_{CM} - \vec{r}_0) \times \vec{V} + I_0 \vec{\omega}$

for stivt legeme med sylindersymmetri om rotasjonsaksen  
refleksjons-



$$\vec{r}_i = \vec{R}_{CM} + \vec{s}_i$$

$$\vec{u}_i = \vec{V} + \vec{u}_i$$

$$\vec{s}_i = \vec{z}_i + \vec{\rho}_i$$

$$\vec{u}_i = \vec{\omega} \times \vec{\rho}_i = \vec{\omega} \times \vec{s}_i \quad (\text{siden } \vec{\omega} \times \vec{z}_i = 0)$$

$$\begin{aligned} \vec{L} &= \sum_i m_i (\vec{r}_i - \vec{r}_0) \times \vec{u}_i \\ &= \sum_i m_i (\vec{R}_{CM} - \vec{r}_0 + \vec{s}_i) \times (\vec{V} + \vec{u}_i) \\ &= \sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{V} + \sum_i m_i \vec{s}_i \times \vec{V} \\ &\quad + \sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{u}_i + \sum_i m_i \vec{s}_i \times \vec{u}_i \end{aligned}$$

1. sum:  $\sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{V} = M (\vec{R}_{CM} - \vec{r}_0) \times \vec{V} = \vec{L}_b$

2. sum:  $\sum_i m_i \vec{s}_i \times \vec{V} = \sum_i m_i (\vec{r}_i - \vec{R}_{CM}) \times \vec{V} = (M\vec{R}_{CM} - M\vec{R}_{CM}) \times \vec{V} = 0$

3. sum:  $\begin{aligned} \sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{u}_i &= \sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times (\vec{\omega} \times \vec{s}_i) \\ &= (\vec{R}_{CM} - \vec{r}_0) \times (\vec{\omega} \times \underbrace{\sum_i m_i \vec{s}_i}_{=0}) = 0 \end{aligned}$

4. sum:  $\sum_i m_i \vec{s}_i \times \vec{u}_i =$  dreieimpuls pga masselementenes bevegelse relativt CM

(Til nå: Helt generelt!)

Stivt lagema:  $\vec{u}_i = \vec{\omega} \times \vec{s}_i$

$$\Rightarrow \sum_i m_i \vec{s}_i \times \vec{u}_i = \sum_i m_i \vec{s}_i \times (\vec{\omega} \times \vec{s}_i)$$

Vektoridentitet:  $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B})$

$$\Rightarrow \sum_i m_i \vec{s}_i \times (\vec{\omega} \times \vec{s}_i) = \sum_i m_i \left\{ \vec{\omega} s_i^2 - \vec{s}_i (\vec{s}_i \cdot \vec{\omega}) \right\}$$

$$= \sum_i m_i \left\{ \vec{\omega} (z_i^2 + \rho_i^2) - (z_i + \vec{\rho}_i) z_i \vec{\omega} \right\}$$

$$= \sum_i m_i \left\{ \vec{\omega} (z_i^2 + \rho_i^2) - z_i^2 \vec{\omega} - z_i \omega \vec{\rho}_i \right\}$$

$$= \sum_i m_i \rho_i^2 \vec{\omega} - \omega \sum_i m_i z_i \vec{\rho}_i$$

$$= \underbrace{I_0}_{I_0} \vec{\omega} - \omega \sum_i m_i z_i \vec{\rho}_i$$

Med refleksions-  
symmetri om  $\hat{\omega}$ , dvs om  $\hat{z}$ :

$$\sum_i m_i z_i \vec{\rho}_i = \sum_i m_i z_i (x_i \hat{x} + y_i \hat{y}) = 0$$

fordi bidraget fra  $(x_i, y_i)$  kanselleres av  
bidraget fra  $(-x_i, -y_i)$  !

Dermed:

$$\vec{L} = \vec{L}_b + \vec{L}_s = M (\vec{R}_{CM} - \vec{r}_0) \times \vec{V} + I_0 \vec{\omega} \quad \text{qed}$$