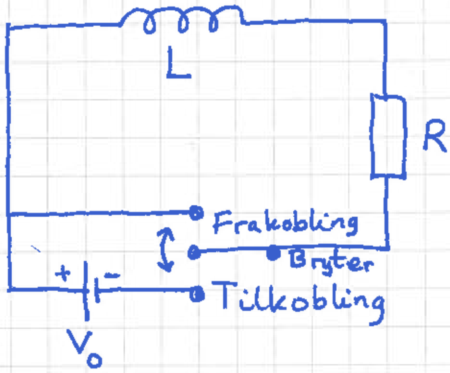
Elektriske kretser; DC og AC; R, L og C

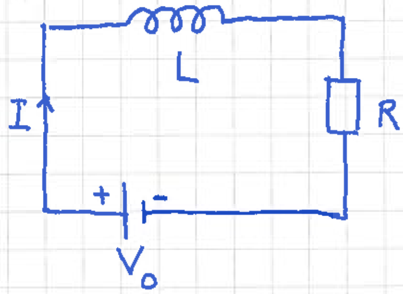
[TM 28.8, 29.1+2+4+6; LHL 25.2, 27.1+2+3+5] [YF 30.4+5+6, 31.5]

① RL-krets; DC

[DC = direct current = likestrøm]



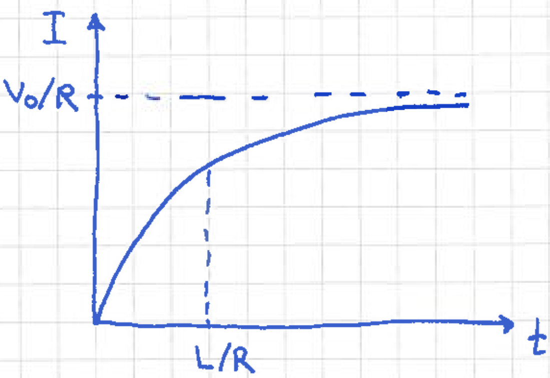
(a) Tilkobling av V_0 ved $t = 0$:



$$K2 \Rightarrow \quad V_0 - L\frac{dI}{dt} - RI = 0$$

Samme ligning, her for I, som for Q i RC-krets s 102, da med $Q(0) = 0$, her med $I(0) = 0$ $\Rightarrow$ Samme løsning!

$$\Rightarrow \quad I(t) = \frac{V_0}{R}\left(1 - e^{-R \cdot t / L}\right)$$

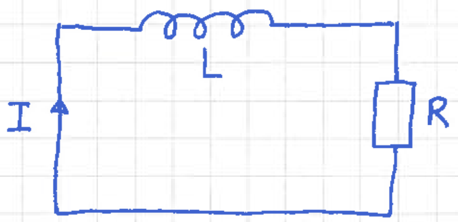


RL-kretsens tidskonstant:

$$\tau = L/R$$

[Jf. $\tau = RC$ for RC-krets]

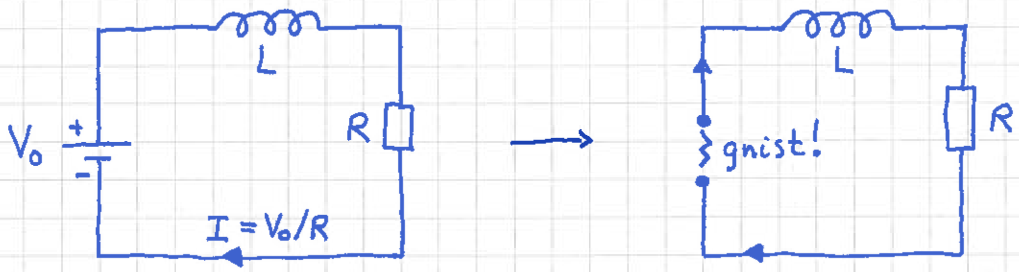
(b) Frakobling av V_0 (etter lang tid $\hat{=}$ nytt $t=0$):



$$K2 \Rightarrow -L\frac{dI}{dt} - RI = 0\,;\quad I(0) = \frac{V_0}{R}$$

$$\Rightarrow I(t) = \frac{V_0}{R}\, e^{-t/\tau}\,;\quad \tau = L/R$$

(c) "Dra ut støpselet"



Strøm I raskt fra V_0/R til 0

$\Rightarrow$ stor $|dI/dt|$ $\Rightarrow$ stor indusert spenning $|L\, dI/dt|$

(selv med liten L) $\Rightarrow$ kortvarig strøm over luftgapet

(selv om motstanden der er stor) $\Rightarrow$ gnist! ("overslag")

AC-kretser [AC = alternating current = vekselstrøm]

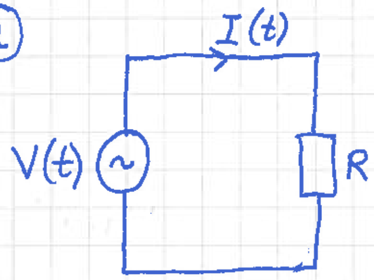
AC spenningskilde:

$$V(t) = V_0 \cos \omega t$$

Frekvens: $f = \frac{\omega}{2\pi}$ [Strømnettet her: $f = 50$ Hz]

[$V_0 = 311$ V ; $V_{rms} = \frac{V_0}{\sqrt{2}} = 220$ V]

(2)



K2 $\Rightarrow V_0 \cos \omega t - RI(t) = 0$

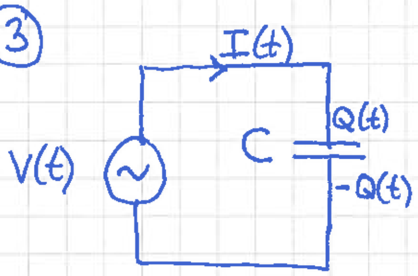
$\Rightarrow I(t) = \frac{V_0}{R} \cos \omega t$

Dvs V(t) og I(t) svinger <u>i fase</u>.

Strømmens amplitude: $I_0 = V_0 / R$

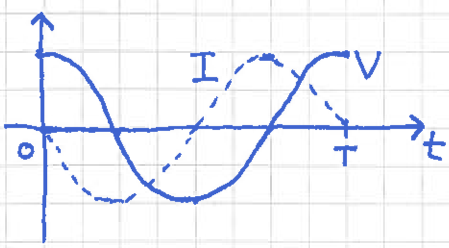
(uavh. av ω)

(3)



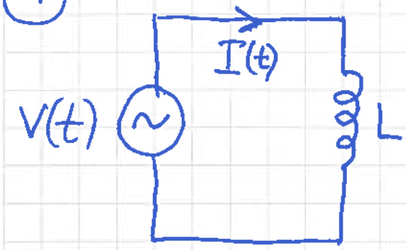
K2 $\Rightarrow V_0 \cos \omega t - \frac{Q(t)}{C} = 0 \;\Rightarrow\; Q(t) = V_0 C \cos \omega t$ [V(t) og Q(t) i fase]

$\Rightarrow I(t) = dQ(t)/dt = -V_0 \omega C \sin \omega t = V_0 \omega C \cos(\omega t + \pi/2)$



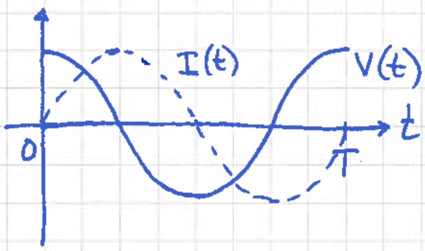
Dvs faseforskjell $\pi/2$ mellom V(t) og I(t); $I_0(\omega) = V_0 \omega C$ <u>øker</u> med økende frekvens

4



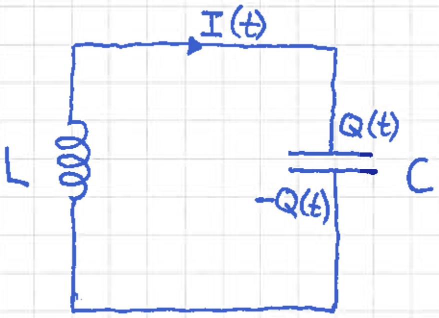
$$K2 \Rightarrow V_0 \cos \omega t - L \frac{dI}{dt} = 0$$

$$\Rightarrow I(t) = \frac{V_0}{\omega L} \sin \omega t = \frac{V_0}{\omega L} \cos\left(\omega t - \frac{\pi}{2}\right)$$



Dvs faseforskjell $-\pi/2$ mellom $V(t)$ og $I(t)$; $I_0(\omega) = V_0/\omega L$ avtar med økende frekvens

5 LC-krets



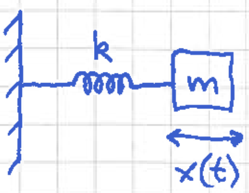
Anta (f.eks.) $Q(0) = Q_0$,

dvs $V_C(0) = Q_0/C$ = spenning over C ved $t = 0$.

$$K2 \Rightarrow -L \frac{dI}{dt} - \frac{Q}{C} = 0 \quad ; \quad I(t) = dQ/dt$$

$$\Rightarrow \ddot{Q} + \frac{1}{LC} Q = 0$$

Enkel harmonisk oscillator, se s. 22 →:



$$\ddot{x} + \frac{k}{m} x = 0$$

$$x(t) = x(0) \cos \omega_0 t \; ; \quad \omega_0 = \sqrt{k/m} \; ; \quad x(0) = x_0$$

$$E = K + U = \tfrac{1}{2} m \dot{x}^2 + \tfrac{1}{2} k x^2 = \tfrac{1}{2} k x_0^2 = \text{konst.} \quad (s. 24)$$

Analoge størrelser i LC-kretsen (se s. 27 B):

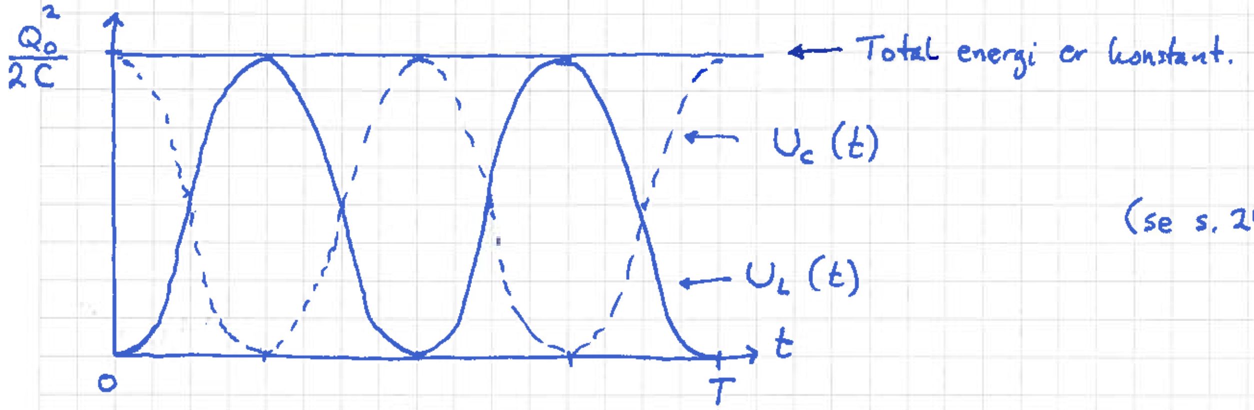
$Q \leftrightarrow x, \quad I \leftrightarrow \dot{x}, \quad L \leftrightarrow m, \quad 1/C \leftrightarrow k, \quad Q_0 \leftrightarrow x_0,$

$\omega_0 = 1/\sqrt{LC} \quad \leftrightarrow \quad \omega_0 = \sqrt{k/m}$

$\Rightarrow Q(t) = Q_0 \cos \omega_0 t, \quad I(t) = -\omega_0 Q_0 \sin \omega_0 t$

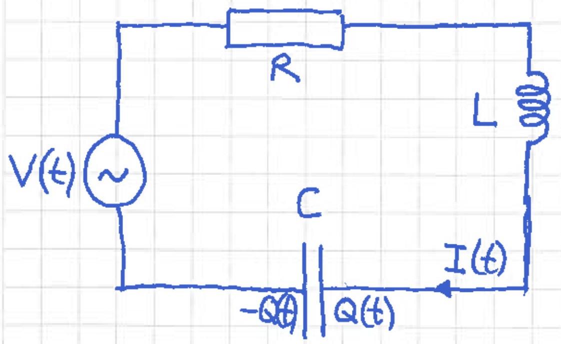
$U_C = \frac{1}{2}\frac{1}{C} Q^2 = \frac{Q_0^2}{2C} \cos^2 \omega_0 t$ (s. 92)

$U_L = \frac{1}{2} L I^2 = \frac{1}{2} L \omega_0^2 Q_0^2 \sin^2 \omega_0 t = \frac{Q_0^2}{2C} \sin^2 \omega_0 t$



(se s. 24)

6 RLC resonanskrets [se s. 26-27B] [og LAB!]

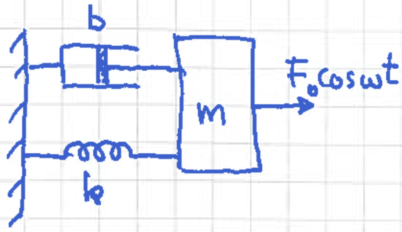


$V(t) = V_0 \cos \omega t$

K2: $V(t) - RI - L\dot{I} - Q/C = 0$

$\Rightarrow L\ddot{Q} + R\dot{Q} + \frac{1}{C} Q = V_0 \cos \omega t$

Mekanisk analogi:



N2: $-kx - b\dot{x} + F_0 \cos \omega t = m\ddot{x}$

$\Rightarrow m\ddot{x} + b\dot{x} + kx = F_0 \cos \omega t$

(dvs $b \leftrightarrow R$)

Fant for mekanisk system:

$$x(t) = A(\omega)\sin(\omega t + \varphi)$$

med $$A(\omega) = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2 + (2\gamma\omega)^2}} \quad ; \quad \omega_0^2 = \frac{k}{m}, \quad 2\gamma = b/m$$

Dermed:

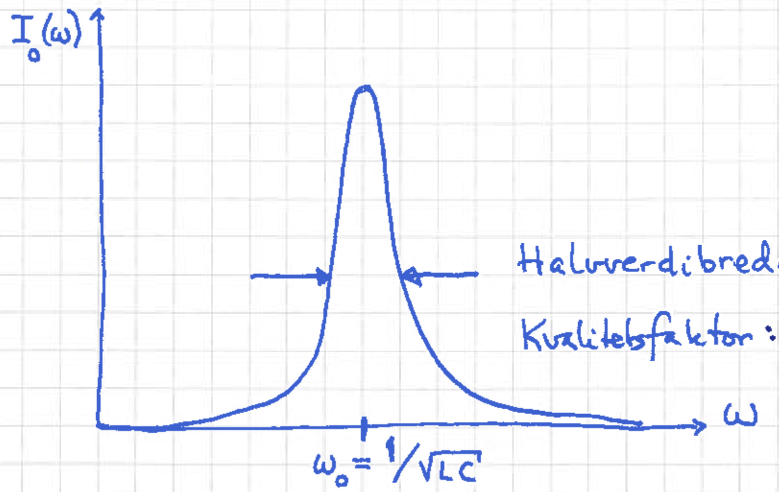
$$Q(t) = Q_0(\omega)\sin(\omega t + \varphi)$$

med $$Q_0(\omega) = \frac{V_0/L}{\sqrt{(\omega^2-\omega_0^2)^2 + (2\gamma\omega)^2}} \quad ; \quad \omega_0^2 = \frac{1}{LC}, \quad 2\gamma = R/L$$

Dvs: $I(t) = \dot{Q}(t) = \omega Q_0(\omega)\cos(\omega t+\varphi) = I_0(\omega)\cos(\omega t+\varphi)$

med strømamplitude

$$\underline{\underline{I_0(\omega) = \omega Q_0(\omega) = \frac{\omega V_0/L}{\sqrt{\left(\omega^2 - \frac{1}{LC}\right)^2 + (R\omega/L)^2}}}}$$



Halvverdibredde: $\Delta\omega \approx 2\gamma = R/L$

Kvalitetsfaktor: $\omega_0/\Delta\omega = \sqrt{L/C}\,/R$

Kan nå måle $I_0(\omega)$ ved å måle spenningen V_R over motstanden:

$I(t) = V_R(t)/R = V_{R0}(\omega)\cos(\omega t+\varphi)/R$, dvs $I_0(\omega) = V_{R0}(\omega)/R$.