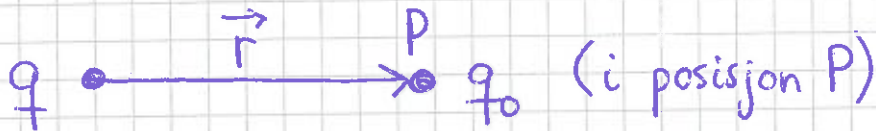


Elektrisk felt [YF 21.4+5 ; LHL 19.4+5]

(64)



elektrisk felt $\stackrel{\text{def}}{=} \text{elektrisk kraft per ladningsenhet}$

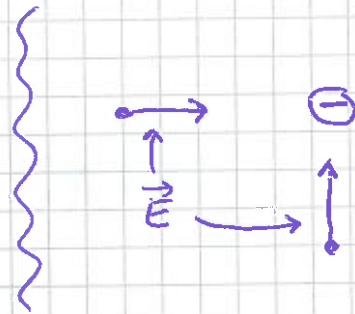
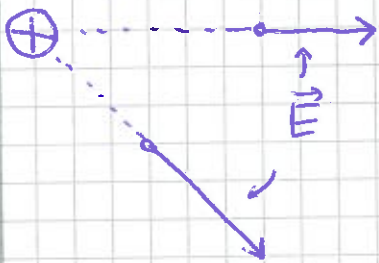
$$\boxed{\vec{E} = \vec{F}/q_0}$$

$$[E] = \text{N/C}$$

Siden \vec{F} er prop. med q_0 , blir \vec{E} uavhengig av q_0 :

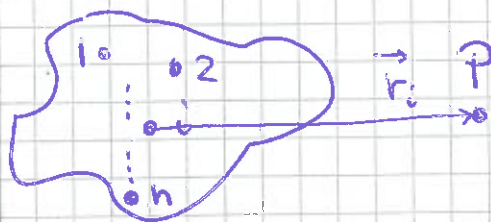
$$\boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}}$$

El. felt fra punktladning q
i avstand r



Dvs: Radieelt bort fra positiv ladning
—||— inn mot negativ —||—

Med flere ladninger (evt. kontinuert ladn. fordeling):



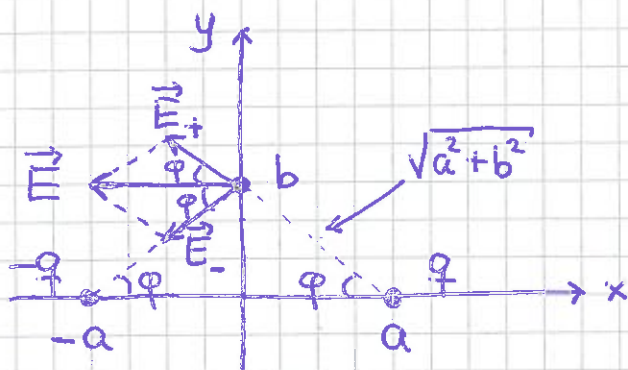
El. felt fra $\{q_1, \dots, q_n\}$ i P :

$$\vec{E} = \sum_i \vec{E}_i = \sum_i \frac{q_i \hat{r}_i}{4\pi\epsilon_0 r_i^2}$$

(dvs SPP gjelder!)

$$\text{(evt. } \vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq_i \hat{r}}{r^2} \text{)}$$

Øks 1: $\pm q$ i $(x,y) = (\pm a, 0)$; finn \vec{E} i $(0,b)$ (65)



$$E_+ = E_- = \frac{q}{4\pi\epsilon_0(a^2+b^2)}$$

Symmetri $\Rightarrow E_y = 0$

$$E_x = E_+^x + E_-^x$$

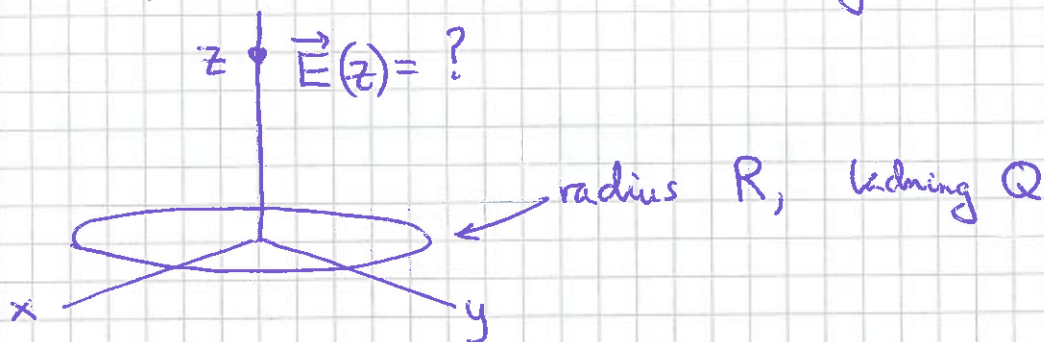
$$E_+^x = E_-^x = E_+ \cos \varphi = \frac{q}{4\pi\epsilon_0(a^2+b^2)} \cdot \frac{a}{\sqrt{a^2+b^2}}$$

$$\Rightarrow \underline{\underline{\vec{E} = -\hat{x} \cdot \frac{2qa}{4\pi\epsilon_0(a^2+b^2)^{3/2}}}}$$

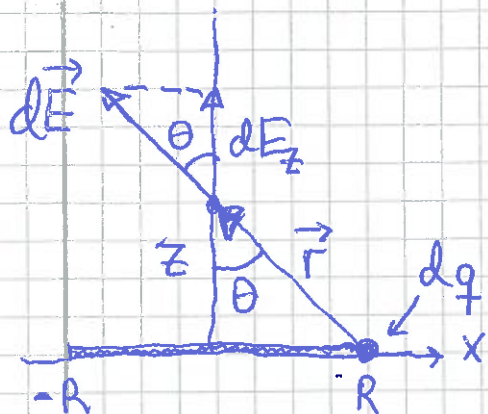
[Langt unna, $b \gg a$, avtar feltstyrken E som $1/b^3$]

[Viktig eksempel; elektrisk dipol; mer om det snart!]

Øks 2: \vec{E} på akse til jevnt ladet ring



Løsning: Symmetri $\Rightarrow E_x = E_y = 0$; $\vec{E}(z) = E_z(z) \hat{z}$



$$d\vec{E} = \frac{dq}{4\pi\epsilon_0 r^2} \hat{r} = dE \cdot \hat{r}$$

$$dE_z = dE \cdot \cos \theta = dE \cdot z/r$$

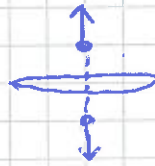
$$r = \sqrt{z^2 + R^2}$$

$$\Rightarrow E_z = \int dE_z = \frac{z}{4\pi\epsilon_0 r^3} \int dq = \underline{\underline{\frac{Qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}}}$$

Kontrollerer om svaret er fornuftig:

(66)

- $[E_z] = [Q/\epsilon_0 z^2]$; OK
- $E_z(0) = 0$; OK
- $E_z \approx Q/4\pi\epsilon_0 z^2$ når $z \gg R$; OK, tilsvarende praktisk tatt en punktladning Q i origo
- $E_z(z) = -E_z(-z)$; OK



Eks 3: \vec{E} på aksen til jevnt ladet sirkular & skive

$$\uparrow dE_z = \frac{dq \cdot z}{4\pi\epsilon_0 (r^2 + z^2)^{3/2}} ; \frac{dq}{Q} = \frac{dA}{A} = \frac{2\pi r \cdot dr}{\pi R^2}$$



$\sigma = Q/\pi R^2 =$ skivas ladn. pr flateenhet

$$E_z = \int dE_z = \int_{r=0}^R \frac{Q \cdot 2\pi r \cdot dr \cdot z}{4\pi\epsilon_0 (r^2 + z^2)^{3/2} \cdot \pi R^2} = \frac{Qz}{2\pi\epsilon_0 R^2} \int_0^R \left(-(r^2 + z^2)^{-1/2} \right)$$

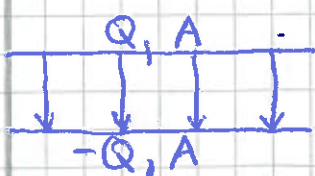
$$= \frac{Qz}{2\pi\epsilon_0 R^2} \left\{ \frac{1}{z} - \frac{1}{\sqrt{R^2 + z^2}} \right\} = \frac{Q}{2\pi\epsilon_0 R^2} \left\{ 1 - \frac{1}{\sqrt{\frac{R^2}{z^2} + 1}} \right\}$$

• $z \gg R \Rightarrow \left(1 + \frac{R^2}{z^2}\right)^{-1/2} \approx 1 - \frac{R^2}{2z^2} \Rightarrow E_z \approx \frac{Q}{4\pi\epsilon_0 z^2}$; OK

• $z \ll R$, dvs svært nær skiva, eller som om skiva var svært stor:

$$E_z \approx \frac{Q}{2\pi\epsilon_0 R^2} = \frac{\sigma}{2\epsilon_0} ; \text{ uavhengig av } z !$$

Anvendelse: Parallellplatekondensator



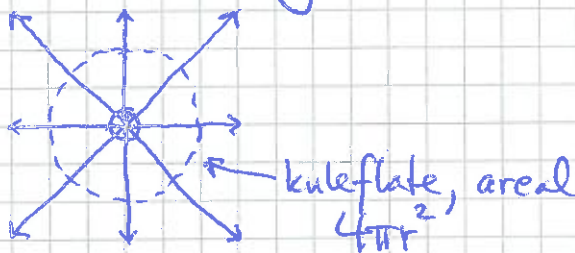
El. feltstyrke mellom platene:

$$E = 2 \cdot \sigma / 2\epsilon_0 = \underline{\underline{\sigma / \epsilon_0}} \quad (E \approx 0 \text{ utenfor})$$

Feltlinjer for \vec{E} [YF 21.6 ; LHL 19.6] (67)

- Visuell framstilling av \vec{E} i et område
- $\vec{E} \parallel$ feltlinjene
- Feltstyrken $E = |\vec{E}|$ proporsjonal med tettheten av feltlinjer, dvs antall feltlinjer som krysser en flate, pr flateenhet, $E \sim N/A$

Eks 1: Punktladning



N feltlinjer ut (inn)
gjennom kuleflaten når
 $q > 0$ ($q < 0$)

Feltlinjetetthet på kuleflaten:

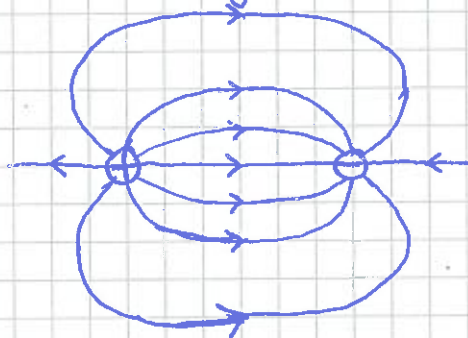
$$\frac{N}{A} = \frac{N}{4\pi r^2} \sim \frac{1}{r^2}$$

Feltstyrke på kuleflaten:

$$E = \frac{q}{4\pi\epsilon_0 r^2} \sim \frac{1}{r^2}$$

$\Rightarrow E$ prop. med $\frac{N}{A}$, OK!

Eks 2: To ladninger med motsatt fortegn (= elektrisk dipol)

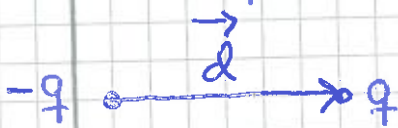


Vi ser at feltlinjer starter på positive ladninger og ender på negative ladninger (eventuelt ∞ langt borte!)

Elektrisk dipol og dipolmoment [YF 21.7; LHL 19.10] (68)

- De fleste molekyler er elektriske dipoler (H_2O , HCl , ...)
 - Hvis ikke, blir de dipoler i et ytre elektrisk felt. Gjelder også gasser, væsker, faste stoffer og enkeltatomer
- ⇒ Viktig for å forstå materialers elektriske egenskaper.

Enkel dipol:



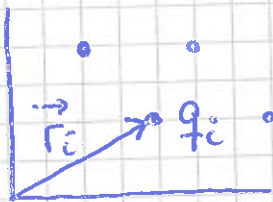
Dipolmoment:

$$\vec{p} = q \vec{d}$$

$$[p] = C \cdot m$$

Merk: Netto ladning alltid null for elektrisk dipol.

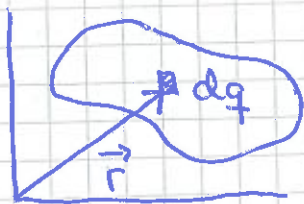
Flere punktladninger:



$$\vec{p} = \sum_i q_i \vec{r}_i$$

$$\left(\sum_i q_i = 0 \right)$$

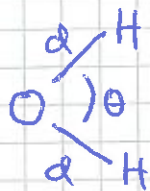
Kontinuerlig ladningsfordeling:



$$\vec{p} = \int \vec{r} dq$$

$$\left(\int dq = 0 \right)$$

Eks 1:

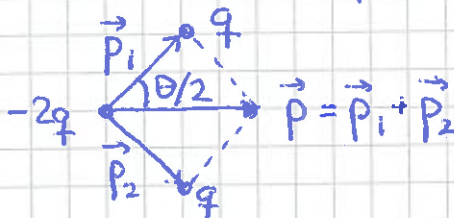


$$d = 0.96 \text{ \AA}$$

$$\theta = 104.5^\circ$$

→ Modell med punktladn:

(69)



$$\Rightarrow p = 2 q d \cos \frac{\theta}{2}$$

Exp. er $p = 6.2 \cdot 10^{-30} \text{ Cm} \Rightarrow q \approx 0.33e$ i vår modell

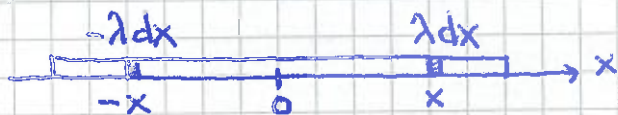
Eks 2:



$\pm \lambda =$ ladning pr lengdeenhet

$$[\lambda] = \text{C/m}$$

Lite ladningspar $\pm dq = \pm \lambda dx$ i innbyrdes avstand $2x$ har dipolmoment $d\vec{p} = \lambda dx \cdot 2x \cdot \hat{x}$:



⇒ Totalt dipolmoment blir:

$$\vec{p} = \int d\vec{p} = \int_0^{L/2} \lambda dx \cdot 2x \hat{x} = 2\lambda \hat{x} \int_0^{L/2} \frac{1}{2} x^2 = \underline{\underline{\frac{\lambda L^2}{4} \hat{x}}}$$

- $[\lambda L^2] = \frac{\text{C}}{\text{m}} \cdot \text{m}^2 = \text{C} \cdot \text{m}$; OK

- som om punktladn. $\pm \lambda \cdot \frac{L}{2}$ var plassert i $x = \pm \frac{L}{4}$,
dvs med innbyrdes avstand $L/2$; OK

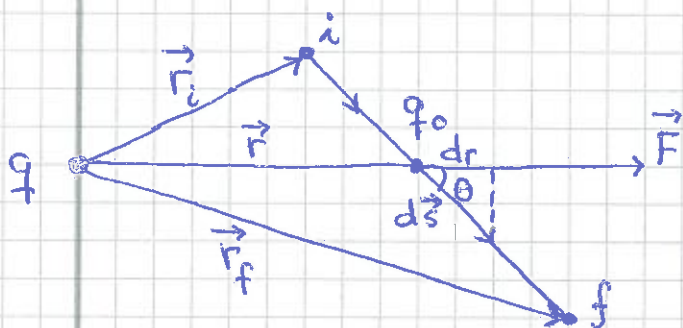
Elektrisk potensial

[YF 23.2; LHL 19.9]

(70)

Coulombkraften \vec{F} er åpenbart konservativ (samme form som gravitasjonskraften).

Potensiell energi for testladning q_0 i \vec{E} -felt fra referanseladning q :



$$\vec{F} \cdot d\vec{s} = F \cdot ds \cdot \cos \theta = F \cdot dr$$

$$\begin{aligned} \Delta U &= U_f - U_i \stackrel{\text{def}}{=} - \int_i^f \vec{F} \cdot d\vec{s} = - \int_{r_i}^{r_f} F \cdot dr \\ &= - \frac{q q_0}{4\pi \epsilon_0} \int_{r_i}^{r_f} \frac{dr}{r^2} \\ &= \underbrace{\frac{q q_0}{4\pi \epsilon_0 r_f}}_{U_f} - \underbrace{\frac{q q_0}{4\pi \epsilon_0 r_i}}_{U_i} \end{aligned}$$

Vi velger $U = 0$ for $r \rightarrow \infty$.

Da er potensiell energi for ladningsparet q og q_0 i innbyrdes avstand r :

$$U(r) = \frac{q q_0}{4\pi \epsilon_0 r}$$

Elektrisk potensial $\stackrel{\text{def}}{=} \text{pot. energi pr ledningsenhet}$

(71)

$$V = U/q_0$$

$$\text{Enhet: } [V] = \frac{J}{C} = V \text{ (volt)}$$

⇒ Punktladning q omgir seg med Coulombpotensialet

$$V(r) = \frac{q}{4\pi\epsilon_0 r}$$

Med $\vec{E} = \vec{F}/q_0$, $V = U/q_0$ og $\Delta U = -\int_i^f \vec{F} \cdot d\vec{s}$ blir

$$\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}$$

Potensialforskjell $V_f - V_i$ mellom posisjon f (final) og i (initial)

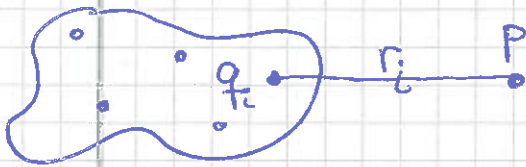
Alternativ enhet for \vec{E} : $[E] = V/m$

Alternativ energienhet:

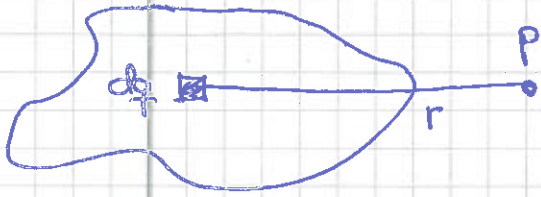
1 eV (elektronvolt) = endringen i potensiell energi når en elementærladning, $q=e$, flyttes fra en posisjon i der potensialet er V_i , til en posisjon f der potensialet er $V_f = V_i + 1V$ (dvs $\Delta V = V_f - V_i = 1V$)

$$\Rightarrow 1 \text{ eV} = \underbrace{1.6 \cdot 10^{-19} \text{ C}}_e \cdot \underbrace{1 \frac{J}{C}}_{1V} = 1.6 \cdot 10^{-19} \text{ J}$$

Potensial fra flere punktladninger, evt. kont. ldn. fordeling: (72)

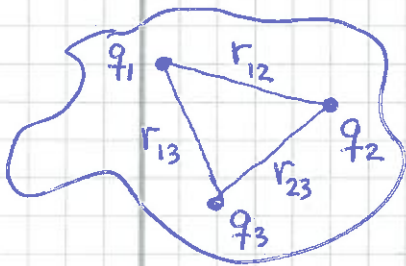


$$V_P = \sum_i V_P^i = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}$$



$$V_P = \int \frac{dq}{4\pi\epsilon_0 r}$$

Potensiell energi for flere punktladninger [YF 23.1; LHL 19.9, 20.3]



Alle ladningene i systemet vekselvirker parvis.

"Referansesystem": $U = 0$ når alle q_i er uendelig langt fra hverandre

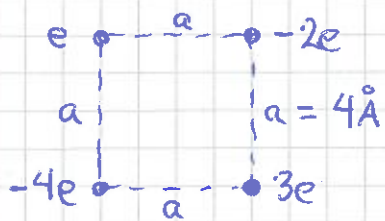
$$\begin{aligned} \Rightarrow U &= U_{12} + U_{13} + \dots + U_{1n} \\ &+ U_{23} + \dots + U_{2n} + \dots + U_{n-1,n} \\ &= \sum_{i=1}^n \sum_{\substack{j=1 \\ (j \neq i)}}^n U_{ij} = \sum_{i < j} U_{ij} = \sum_{i < j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} \end{aligned}$$

Eks 1: Hva er hvileenergien mc^2 til et elektron? et proton?

$$\begin{aligned} \text{Løsning: } m_e c^2 &= 9.11 \cdot 10^{-31} \text{ kg} \cdot (3 \cdot 10^8 \text{ m/s})^2 = 8.20 \cdot 10^{-14} \text{ J} \cdot \frac{1}{1.6 \cdot 10^{-19} \text{ J/eV}} \\ &= 5.1 \cdot 10^5 \text{ eV} \approx \underline{\underline{0.5 \text{ MeV}}} \end{aligned}$$

$$m_p c^2 = 1.67 \cdot 10^{-27} \text{ kg} \cdot (3 \cdot 10^8 \text{ m/s})^2 = (1.6 \cdot 10^{-19})^{-1} \frac{\text{eV}}{\text{J}} \approx \underline{\underline{940 \text{ MeV}}}$$

Eks 2:



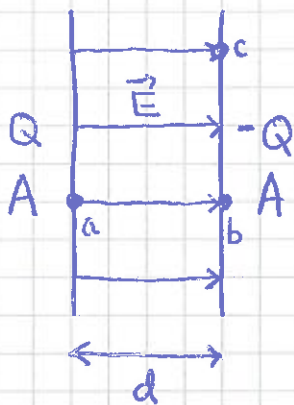
$$U = ?$$

Løsning:

$$U = \frac{e^2}{4\pi\epsilon_0 a} \left\{ 1 \cdot (-2) + (-2) \cdot 3 + 3 \cdot (-4) + (-4) \cdot 1 + \frac{1}{\sqrt{2}} \cdot 1 \cdot 3 + \frac{1}{\sqrt{2}} \cdot (-2) \cdot (-4) \right\}$$

$$= \frac{e^2}{4\pi\epsilon_0 a} \left\{ -24 + \frac{11}{\sqrt{2}} \right\}$$

$$= 9 \cdot 10^9 \cdot \frac{1.6 \cdot 10^{-19}}{4 \cdot 10^{-10}} \cdot (-16.22) \text{ eV} \approx \underline{\underline{-58 \text{ eV}}}$$

Eks 3: ΔV i uniformt \vec{E} -felt

$$V_a - V_b = - \int_b^a \vec{E} \cdot d\vec{s} = \int_a^b \vec{E} \cdot d\vec{s} = \underline{\underline{E \cdot d}}$$

$$V_c - V_b = - \int_b^c \vec{E} \cdot d\vec{s} = \underline{\underline{0}} \quad (\vec{E} \perp d\vec{s})$$

Anta metallplater, $A = 10 \text{ cm}^2$, $d = 20 \text{ mm}$, $Q = 2.0 \mu\text{C}$.Da har vi $E = \sigma/\epsilon_0 = Q/A\epsilon_0$ og en potensialforskjell mellom platen

$$\Delta V = V_a - V_b = E \cdot d = \frac{Q \cdot d}{A \cdot \epsilon_0} = \frac{2.0 \cdot 10^{-6} \text{ C} \cdot 2.0 \cdot 10^{-3} \text{ m}}{10 \cdot 10^{-4} \text{ m}^2 \cdot \epsilon_0}$$

$$\frac{1}{\epsilon_0} = 4\pi \cdot \frac{1}{4\pi\epsilon_0} = 4\pi \cdot 9 \cdot 10^9 \frac{\text{Vm}}{\text{C}}$$

$$\Rightarrow \Delta V = 4.0 \cdot 10^{-6-3+3} \cdot 4\pi \cdot 9 \cdot 10^9 \text{ V} = 144\pi \cdot 10^3 \text{ V}$$

$$\approx \underline{\underline{450 \text{ kV}}}$$