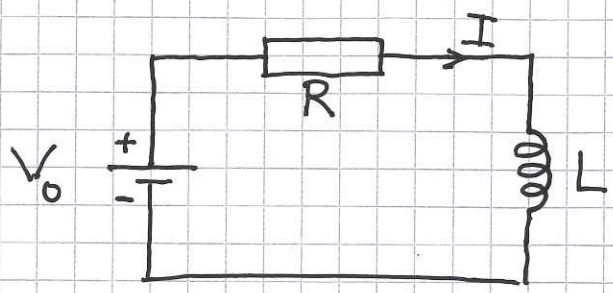


Kretser og anvendelser ; DC og AC

[OS2 14, 15 ; YF 30.4-6 ; LHL 25.2, 27.1+2+3+5]
31.5

① RL-krets ; DC



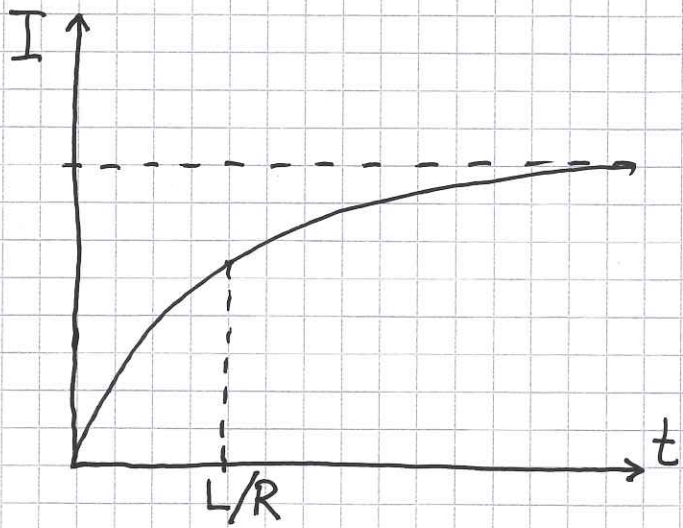
V_0 kobles til ved $t=0$

$$K2: V_0 - RI - L \frac{dI}{dt} = 0$$

det samme ligning for I som for Q i RC-kretsen

$$\Rightarrow I(t) = \frac{V_0}{R} (1 - e^{-t/\tau})$$

med tidskonstant $\tau = L/R$

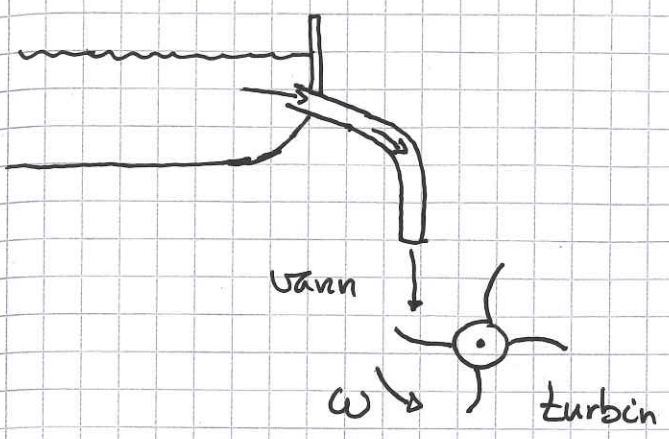


Induktansen L gir en induert motspenning i kretsen når vi med V_0 prøver å øke strømmen. Da blir I ikke umiddelbart V_0/R , det tar litt tid!

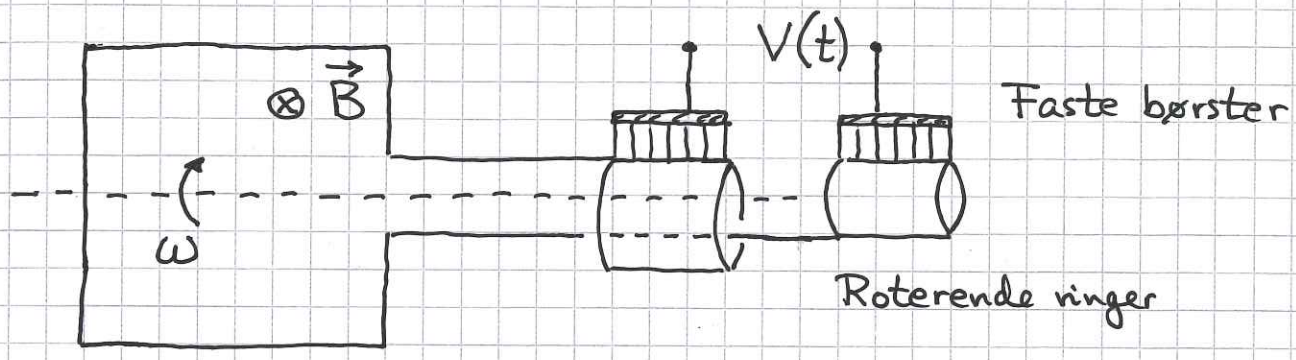
Tidsskala gitt ved $\tau = L/R$.

AC spenningskilde

(AC = alternating current = vekselstrøm)



$U_{\text{vann}} \rightarrow K_{\text{vann}} \rightarrow K_{\text{rot turbin}}$
 som roterer spole i \vec{B} -felt
 og gir harmonisk tidsavhengig
 omsluttet fluks $\phi(t)$

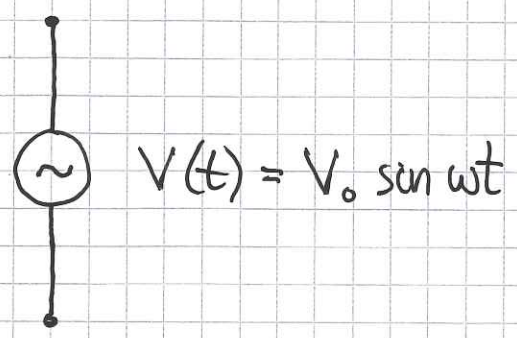


Spole, N viklinger, omsluttet areal A

$$\Rightarrow \phi(t) = NBA \cos \omega t$$

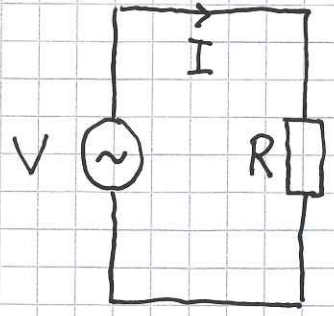
$$\Rightarrow V(t) = V_0 \sin \omega t \quad \text{med} \quad V_0 = NBA\omega$$

Kretssymbol:



Frekvens: $f = \frac{\omega}{2\pi}$
 Europa: 50 Hz

2

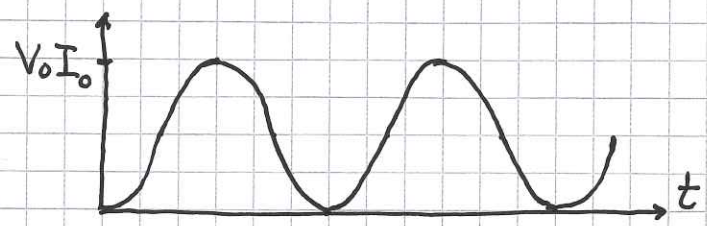


$$K2: V_0 \sin \omega t - RI = 0$$

$$\Rightarrow I(t) = I_0 \sin \omega t ; I_0 = V_0/R$$

$\Rightarrow V$ og I svinger i fase

Effekttap: $P(t) = V(t)I(t) = V_0 I_0 \sin^2 \omega t$



Midlere effekttap: $\langle P \rangle = V_0 I_0 \langle \sin^2 \omega t \rangle = \frac{1}{2} V_0 I_0$

Effektivverdier: $\langle P \rangle = V_{rms} \cdot I_{rms}$
 med $V_{rms} = V_0/\sqrt{2}$; $I_{rms} = I_0/\sqrt{2}$

I veggen: $V_{rms} \approx 230 \text{ V}$; $V_0 \approx 325 \text{ V}$

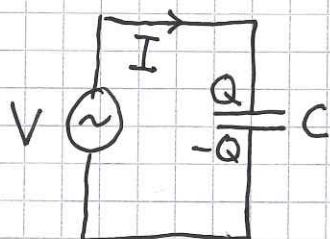
rms = root mean square :

$$V_{rms} = \sqrt{\langle V^2 \rangle}$$

$$= \sqrt{\frac{1}{T} \int_0^T V(t)^2 dt}$$

$$= V_0/\sqrt{2}$$

③



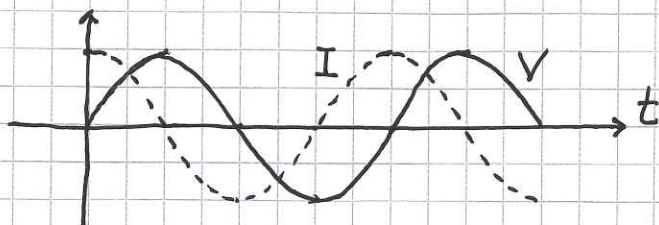
$$K2: V_0 \sin \omega t - \frac{Q}{C} = 0$$

$$\Rightarrow Q(t) = V_0 C \sin \omega t$$

$$\Rightarrow I(t) = V_0 \omega C \cos \omega t$$

⑥9

Faseforskjell $\pi/2$ mellom $V(t)$ og $I(t)$:

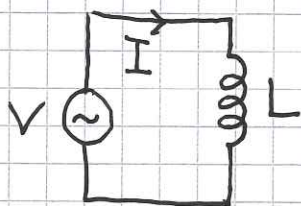


$$\langle P \rangle = V_0 I_0 \langle \sin \omega t \cdot \cos \omega t \rangle = \frac{1}{2} V_0 I_0 \langle \sin 2\omega t \rangle = 0$$

Dvs, kondensatoren er et tapsfritt kretselement.

Strøamplituden $I_0(\omega) = V_0 \omega C$ øker med frekvensen.

④

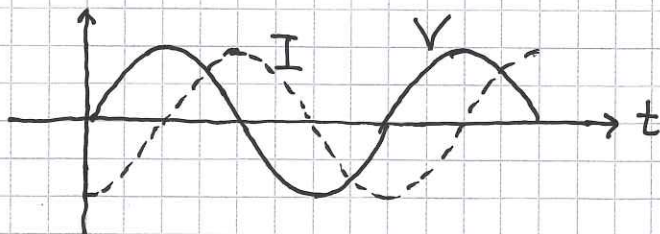


$$K2: V_0 \sin \omega t - L \dot{I} = 0$$

$$\Rightarrow \dot{I}(t) = \frac{V_0}{L} \sin \omega t$$

$$\Rightarrow I(t) = -\frac{V_0}{\omega L} \cos \omega t$$

Faseforskjell $(-)\pi/2$ mellom $V(t)$ og $I(t)$:

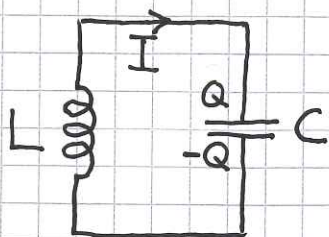


$$\langle P \rangle \sim \langle \sin \omega t \cdot \cos \omega t \rangle = 0$$

Dvs, en induktans er også et tapsfritt kretselement.

Strøamplituden $I_0(\omega) = V_0 / \omega L$ avtar med frekvensen.

⑤ LC-krets



Anta $Q(0) = Q_0$

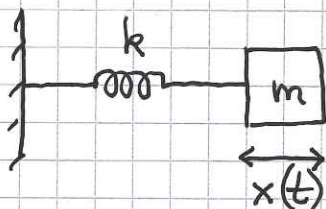
$$K2: -L\dot{I} - Q/C = 0$$

$$I = \dot{Q} \Rightarrow \underline{\underline{\ddot{Q} + \frac{1}{LC} Q = 0}}$$

Dvs: Harmonisk oscillator.

$$\Rightarrow Q(t) = Q_0 \cos \omega_0 t \quad ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Mekanisk analogi:



$$\ddot{x} + \frac{k}{m} x = 0$$

$$x(t) = x_0 \cos \omega_0 t \quad ; \quad \omega_0 = \sqrt{\frac{k}{m}}$$

Analoge størrelser:

$$Q \leftrightarrow x, \quad I \leftrightarrow \dot{x}, \quad L \leftrightarrow m, \quad C \leftrightarrow 1/k$$

$$K = \frac{1}{2} m \dot{x}^2 \leftrightarrow \frac{1}{2} L I^2 = \text{energi i } \vec{B}\text{-felt i spolen}$$

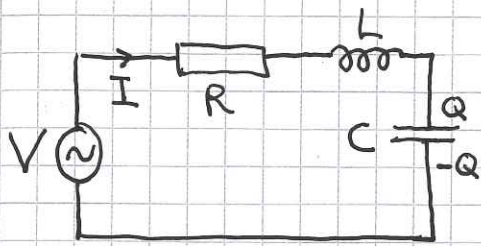
$$U = \frac{1}{2} k x^2 \leftrightarrow \frac{1}{2} \frac{1}{C} Q^2 = \text{---} \vec{E}\text{-felt i kondensatoren}$$

Konservativt system. Ingen dissipasjon av energi når $R = 0$:

$$\begin{aligned} Q^2/2C + \frac{1}{2} L I^2 &= \frac{Q_0^2}{2C} \cos^2 \omega_0 t + \frac{1}{2} L Q_0^2 \underbrace{\omega_0^2}_{=1/LC} \sin^2 \omega_0 t \\ &= \frac{Q_0^2}{2C} = \text{konstant} \end{aligned}$$

⑥ RLC resonanskrets

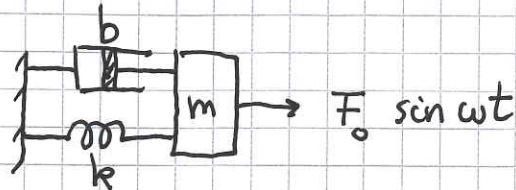
⑦①



$$K2: V_0 \sin \omega t - RI - L\dot{I} - \frac{Q}{C} = 0$$

$$\Rightarrow L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = V_0 \sin \omega t$$

Mekanisk analogi:



$$N2: m\ddot{x} + b\dot{x} + kx = F_0 \sin \omega t$$

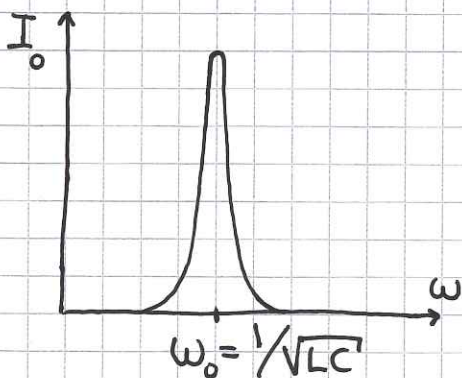
$$\Rightarrow b \leftrightarrow R, \quad F_0 \leftrightarrow V_0$$

\(\Rightarrow\) Resonans när $\omega \approx \omega_0 = 1/\sqrt{LC}$:

$$Q(t) = Q_0(\omega) \sin(\omega t + \varphi)$$

$$Q_0(\omega) = \frac{V_0/L}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\gamma\omega)^2}}; \quad 2\gamma = R/L$$

$$I(t) = I_0(\omega) \cos(\omega t + \varphi); \quad I_0(\omega) = \omega Q_0(\omega)$$



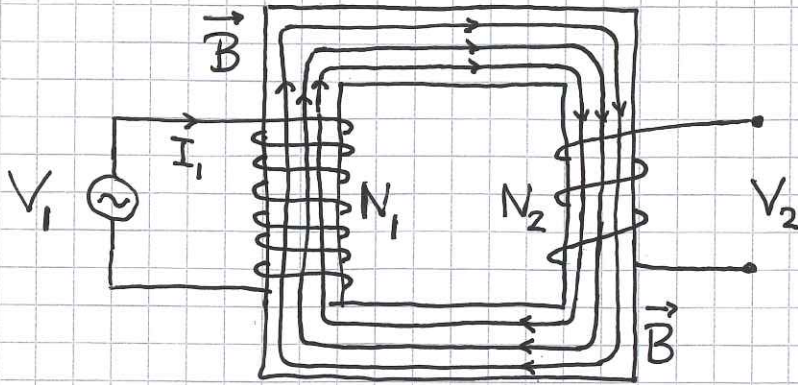
Halvverdbredda: $\Delta\omega = 2\gamma = R/L$

$$\text{Kvalitetsfaktor: } \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} = \frac{\sqrt{L/C}}{R}$$

Måler $I_0(\omega)$ og $Q_0(\omega)$ ved å måle $V_R = RI$ og $V_C = Q/C$ over hhv R og C .

⑦ Transformator

⑦2



Magnetisering av jernkjernen fører til at feltlinjene for \vec{B} følger jernet; gir samme $|\vec{B}|$ i begge spoler.

Omsluttet fluks:

$$\Phi_1 = N_1 AB \quad ; \quad \Phi_2 = N_2 AB$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\dot{\Phi}_1}{\dot{\Phi}_2} = \frac{N_1}{N_2}$$

Dvs: Spenningen inn, V_1 , kan transformeres til spenning ut, V_2 , med redusert eller økt amplitude, med hhv $N_2 < N_1$ eller $N_2 > N_1$:

$$V_2 = \frac{N_2}{N_1} \cdot V_1$$