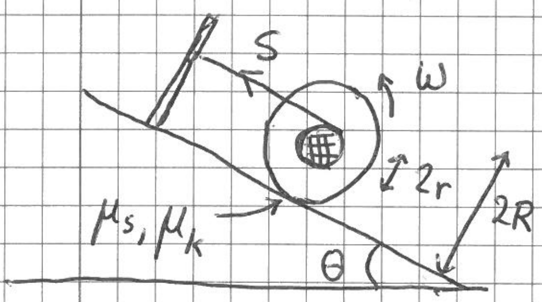
Statisk likevekt [YF 11.1–11.3 ; LL 7.1]

Et stivt legeme forblir i ro $(\vec{p} = 0, \vec{L} = 0)$

bare dersom $\underbrace{\sum_i \vec{F}_i}_{=\text{netto ytre kraft}} = 0$ og $\underbrace{\sum_i \vec{\tau}_i}_{=\text{netto ytre dreiemoment}} = 0$

Ekempler, rotasjon

Eks 1: Snelle på skråplanet



- Ved hvilken vinkel θ_0 begynner snella å gli/rotere?
- Hva er snordraget S og akselerasjonen a når den glir?

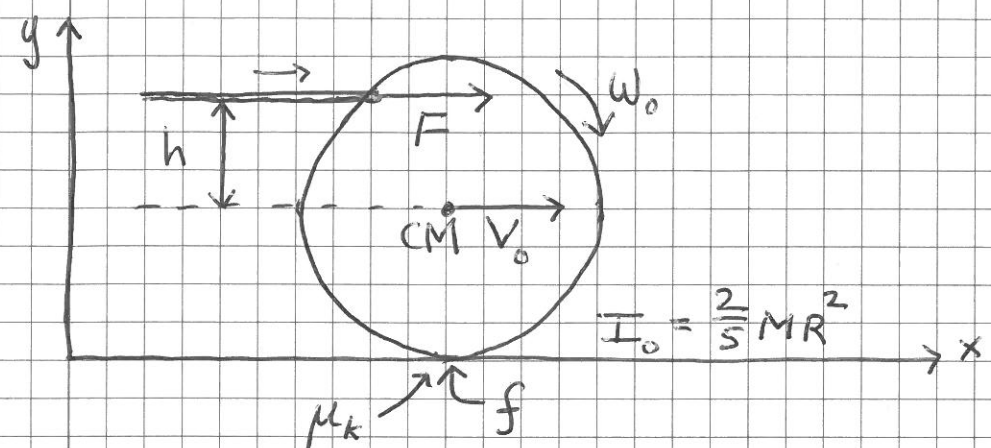
Strategi:

N1 langs skråplanet + N1 for rot. om CM, $f = f_{max} \Rightarrow \theta_0$

N2 —— " —— + N2 —— " ——, $f = \mu_k N \Rightarrow S$ og a

(Øv. 6)

Eks 2: Snooker [LL 6.7]



Kortvarig støt med kraft F, varighet Δt, i høyde h over senterlinjen; $F \gg f$.

Finn kulas bevegelse. (Øv. 6)

N2: $F\Delta t = \Delta p = MV_0$

N2, rot om CM: $\tau\,\Delta t = \Delta L = I_0\,\omega_0$,

med $\tau = F \cdot h$

Stor h $\Rightarrow$ $\omega_0 > V_0/R$

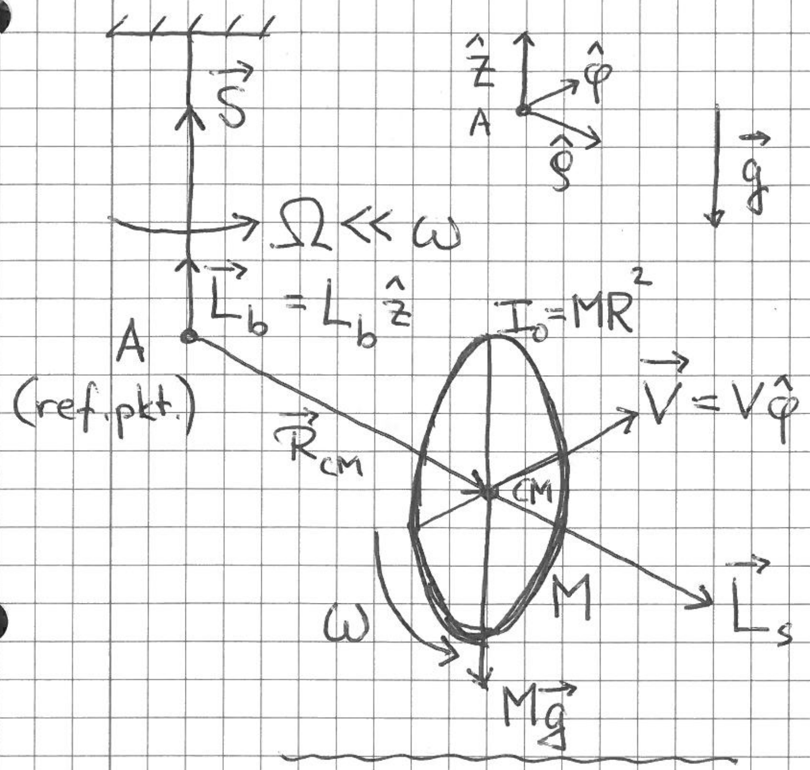
$\Rightarrow$ sluring, med $\vec{f}$ mot venstre

Liten h $\Rightarrow$ omvendt

Passelig $h = h_0$ $\Rightarrow$ $\omega_0 = V_0/R$ $\Rightarrow$ ren rulling fra start!

Etter hvert ren rulling uansett, med $f = 0$.

Eks 3: Presesjon (Gyroskop) [YF 10.7 ; LL 6.10]



Eksperimentelt:

$M = 5\,kg$

$R = 0.3\,m$

$R_{CM} = 0.2\,m$

$T_\Omega = \frac{2\pi}{\Omega} = \underline{4.7\ s}$

$\boxed{\text{Hva er } \omega?}$

Løsning:

Dreieimpuls relativt A:

$$\vec{L}_A = \vec{L}_b + \vec{L}_s = \vec{R}_{CM} \times M\vec{V} + I_0\vec{\omega}$$

$$= R_{CM} M V\ \hat{z} + MR^2 \omega\ \hat{\rho}$$

$$= M R_{CM}^2\, \Omega\ \hat{z} + MR^2\ \omega\ \hat{\rho}$$

$\Rightarrow \vec{L}_A \approx \vec{L}_s$ siden $\Omega \ll \omega$ (og $R_{CM} \approx R$)

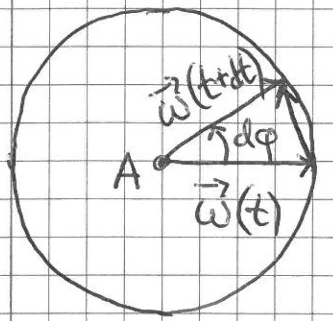
Bare tyngden $M\vec{g} = -Mg\hat{z}$ har dreiemoment relativt A: (Snordraget $\vec{S}$ går gjennom A.)

$$\vec{\tau}_A = \vec{R}_{CM} \times M\vec{g} = R_{CM} M g\,(\hat{\rho} \times (-\hat{z})) = R_{CM} M g\, \hat{\varphi}$$

N2 for rotasjon, med A som ref.punkt (se s 46):

$$\vec{\tau}_A = \frac{d\vec{L}_A}{dt} \approx \frac{d\vec{L}_s}{dt} = I_0 \frac{d\vec{\omega}}{dt} \qquad \left(\frac{d\vec{L}_b}{dt} \approx 0\right)$$

Sett ned langs z-aksen:



$$d\vec{\omega} = \omega \, d\varphi \, \hat{\varphi}$$

$$\Rightarrow \frac{d\vec{\omega}}{dt} = \omega \frac{d\varphi}{dt} \hat{\varphi} = \omega \Omega \hat{\varphi}$$

Dermed: $\underbrace{R_{CM} M g}_{\tau} = I_0 \omega \Omega = M R^2 \omega \Omega$

$$\Rightarrow \underline{\underline{\omega = R_{CM} g / R^2 \Omega}}$$

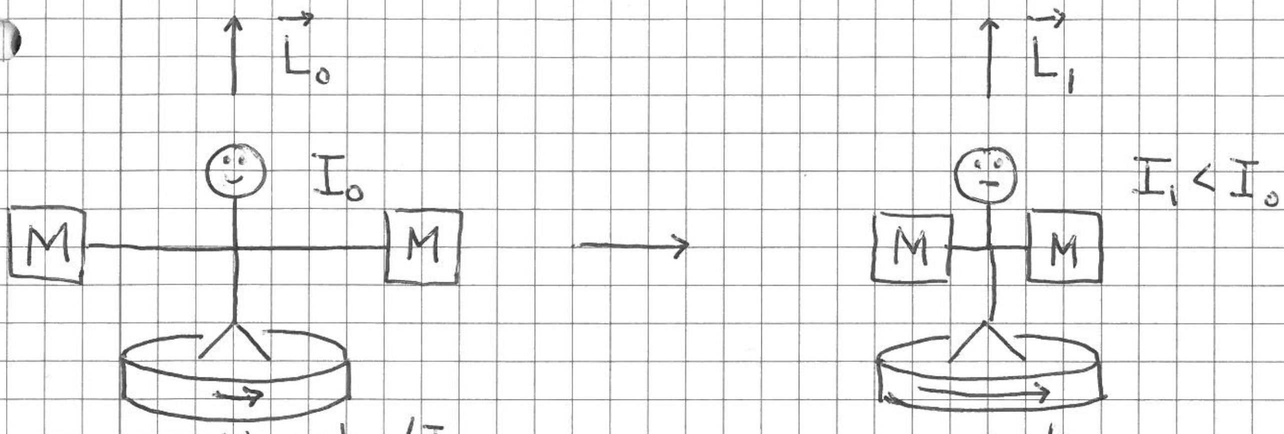
Hjulets omløpstid:

$$\underline{\underline{T_\omega = \frac{2\pi}{\omega} = \frac{(2\pi R)^2}{R_{CM} \, g \, T_\Omega}}}$$

Tallverdi:

$$T_\omega \approx \frac{(2\pi \cdot 0.3)^2}{0.2 \cdot 10 \cdot \underset{\sim}{4.7}} \approx \underline{\underline{0.38 \text{ s}}}$$

Eks 4: Piruett [YF 10.6; LL 6.5]

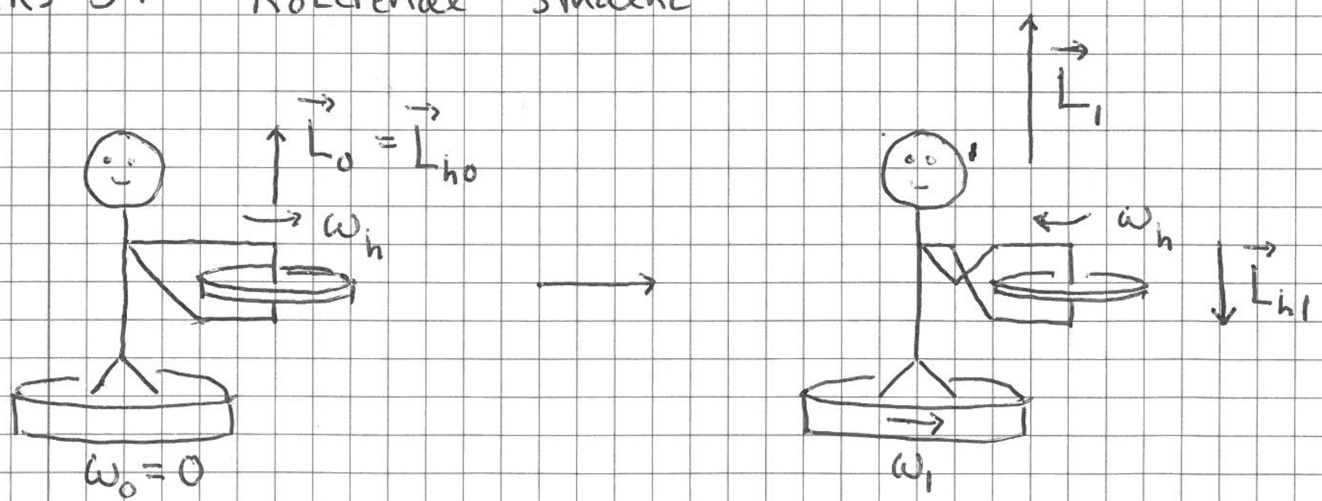


$\omega_0 = L_0 / I_0$ $\qquad\qquad$ $\omega_1 = L_1 / I_1 > \omega_0$

$\vec{\tau}_{ytre} = 0 \Rightarrow \vec{L}_1 = \vec{L}_0 \Rightarrow \omega_1 = L_1 / I_1 = L_0 / I_1 = \underline{\underline{\omega_0 I_0 / I_1}} > \omega_0$

(Men $K_1 > K_0$: Gjør arbeid på "M+M" med kjemisk energi i musklene)

Eks 5: Roterende student



$$\vec{\tau}_{ytre} = 0 \Rightarrow \vec{L}_1 + \underbrace{\vec{L}_{h1}}_{-\vec{L}_0} = \vec{L}_0$$

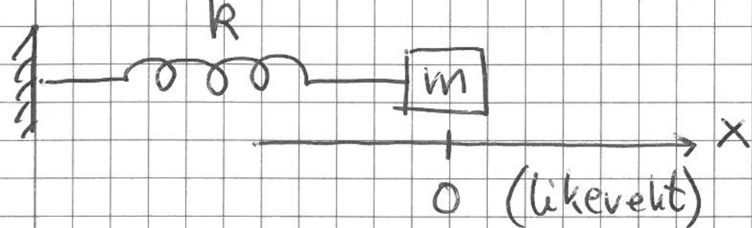
$$\Rightarrow \underline{\underline{\vec{L}_1 = 2\vec{L}_0}}$$

$\{$ SVINGNINGER $\}$ [YF 14 ; LL 9]

= oscillasjoner = periodisk oppførsel omkring likevekt

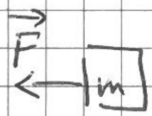
Eks: Masse/fjær. Pendel. Gitarstreng. Luft i blåseinstrument. Atomer i molekyler / krystaller. osv

Harmonisk oscillator [YF 14.2 ; LL 9.1–9.3]

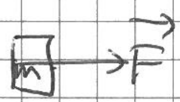


Likevekt ($F=0$) når m i $x=0$

m i $x>0 \Rightarrow$ strukket fjær $\Rightarrow \vec{F} \sim -\hat{x}$



m i $x<0 \Rightarrow$ sammenpresset fjær $\Rightarrow \vec{F} \sim +\hat{x}$



Hookes lov (ideell fjær): $|\vec{F}| \sim |x|$

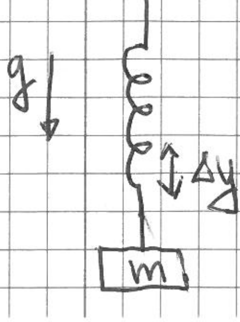
$$\Rightarrow \boxed{\vec{F} = -kx\hat{x}}$$

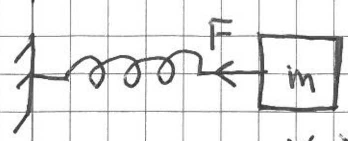
k = fjærkonstant ; $[k] = \mathrm{N/m}$

Vertikalt: Fjæra strekkes Δy når m henges på.

N1 (i likevekt) $\Rightarrow mg = k\Delta y$

$\Rightarrow \underline{\Delta y = mg/k}$





$x > 0$

N2: $-kx = m\ddot{x}$

$$\Downarrow$$

$$\ddot{x} + \frac{k}{m}x = 0$$

Innfører $\omega_0 \equiv \sqrt{k/m}$ $\Longrightarrow$ $\boxed{\ddot{x} + \omega_0^2 x = 0}$ Harm. osc. i 1D

Løsning: Ser at $\sin \omega_0 t$ og $\cos \omega_0 t$ løser ligningen

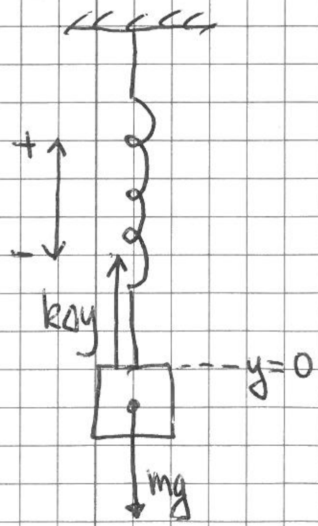
$\Rightarrow$ Generell løsning:

$$x(t) = B\cos\omega_0 t + C\sin\omega_0 t \quad \text{evt} \quad x(t) = A\cos(\omega_0 t + \varphi)$$

der konstanter B, C evt A, φ fastlegges med 2 initialbetingelser, f.eks. $x(0) = x_0$ og $\dot{x}(0) = v_0$

[Se øving for sammenheng mellom B, C og A, φ]

Samme ligning for vertikal bevegelse om "strukket likevekt" med $\Delta y = mg/k$:



N2 med $y \neq 0$:

$$m\ddot{y} = \Sigma F = -mg + k(\Delta y - y)$$

$$= -mg + k \cdot \frac{mg}{k} - k \cdot y$$

$$= -ky$$

$$\Rightarrow \ddot{y} + \omega_0^2 y = 0$$

med $\omega_0^2 = k/m$

Diverse størrelser og begreper:

$$x(t) = A\cos(\omega_0 t + \varphi)$$

A = amplitude = max utsving fra likevekt $[A] = [x]$

ω_0 = vinkelfrekvens $[\omega_0] = 1/s$

$T = 2\pi/\omega_0$ = periode = tid pr hel svingning $[T] = s$

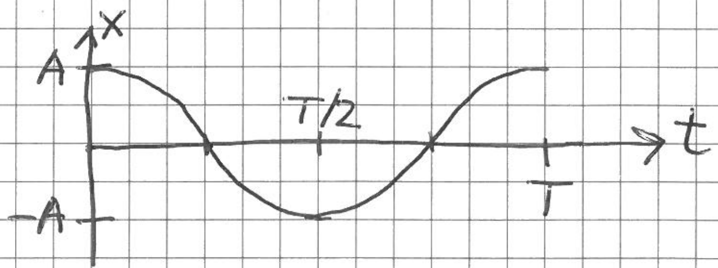
$f = 1/T$ = frekvens = svingninger pr tidsenhet $[f] = \frac{1}{s} = Hz$

$\omega_0 t + \varphi$ = svingningens fase

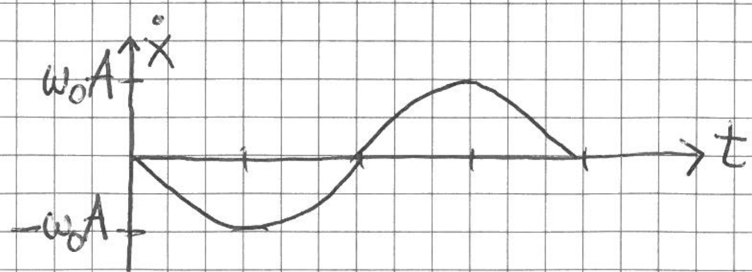
φ = fasekonstant $[\varphi] = 1$

Anta $\varphi = 0$ og $A > 0$:

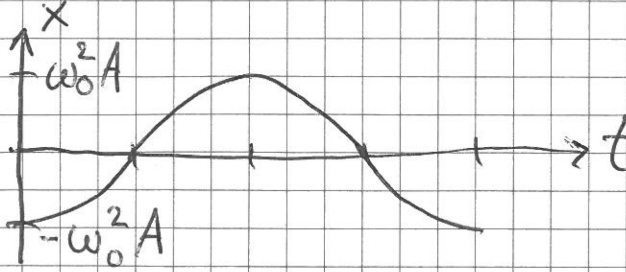
$$x(t) = A\cos\omega_0 t$$



$$\dot{x}(t) = -\omega_0 A\sin\omega_0 t$$
$$= \omega_0 A\cos\left(\omega_0 t + \frac{\pi}{2}\right)$$



$$\ddot{x}(t) = -\omega_0^2 A\cos\omega_0 t$$
$$= -\omega_0^2 x(t)$$
$$= \omega_0^2 A\cos(\omega_0 t + \pi)$$



Energi i harmonisk oscillator [YF 14.3; LL 9.4] 58

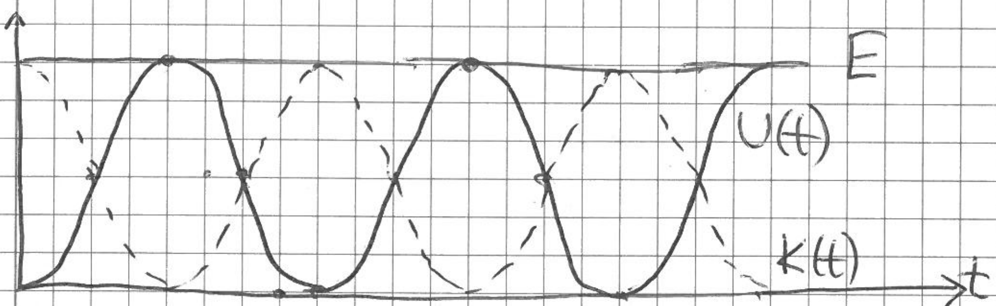
$K(t) = \frac{1}{2} m \dot{x}(t)^2 = \frac{1}{2} m \omega_0^2 A^2 \sin^2 \omega_0 t = \frac{1}{2} k A^2 \sin^2 \omega_0 t$

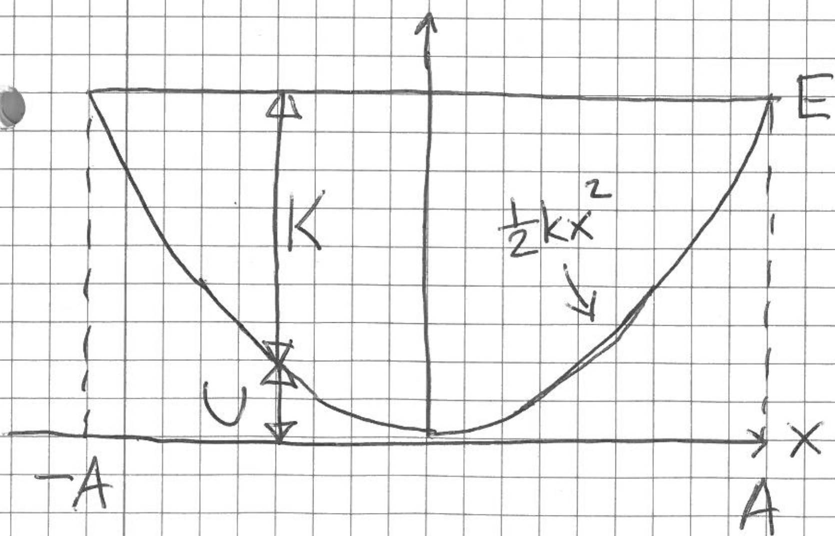
$U = -\int_0^x F(x)\,dx = -\int_0^x (-kx)\,dx = \frac{1}{2} k x^2$

$\Rightarrow U(t) = \frac{1}{2} k x(t)^2 = \frac{1}{2} k A^2 \cos^2 \omega_0 t$

$\Rightarrow E = K + U = \frac{1}{2} k A^2 =$ konstant (tidsuavhengig)

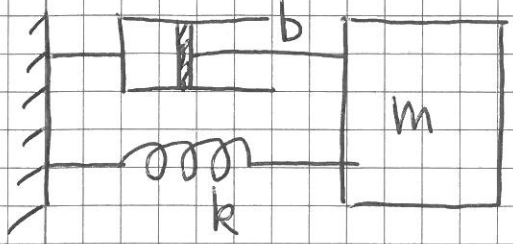
$\Rightarrow$ Systemet er konservativt; mekanisk energi E er bevart





Dempet fri svingning [YF 14.7 ; LL 9.7]

- Anta $f = -b\dot{x}$ (dvs langsom bevegelse i fluid)



N2: $-kx - b\dot{x} = m\ddot{x}$

$$\Rightarrow \ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$$

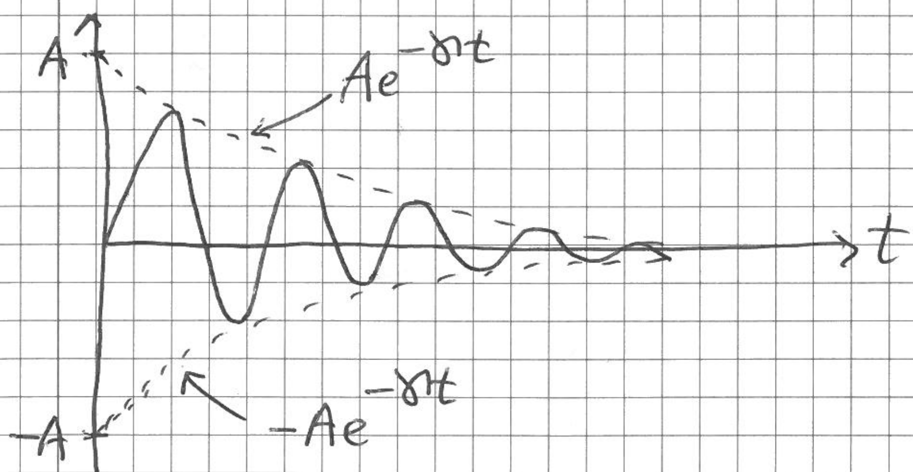
$\gamma \equiv b/2m$, $\omega_0^2 \equiv k/m$

$[\gamma] = [\omega_0] = 1/s$

Form på løsning avhenger av γ/ω_0 :

Underkritisk (svak) demping, $\gamma < \omega_0$:

$$x(t) = Ae^{-\gamma t}\sin(\omega t + \varphi) \; ; \; \omega = \sqrt{\omega_0^2 - \gamma^2}$$



Dempet svingebevegelse med eksponentielt avtagende amplitude $Ae^{-\gamma t}$.

Overkritisk demping, $\gamma > \omega_0$

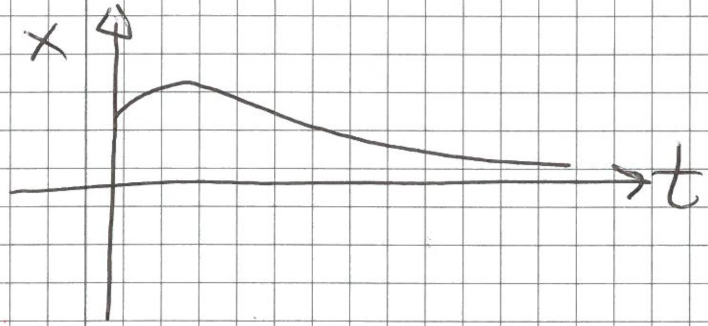
$$x(t) = Ae^{-\alpha_1 t} + Be^{-\alpha_2 t}$$

$$\alpha_1 = \gamma + \sqrt{\gamma^2 - \omega_0^2} \; ; \; \alpha_2 = \gamma - \sqrt{\gamma^2 - \omega_0^2}$$

Kritisk demping, $\gamma = \omega_0$

$$x(t) = Ae^{-\gamma t} + Bte^{-\gamma t}$$

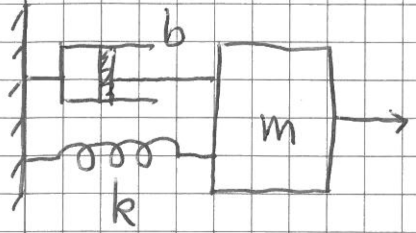
Dvs, $\gamma \geq \omega_0$ gir ikke svingninger, men eksponentielt avtagende utsving $x(t)$:



(Her: $x(0) > 0$
$\dot{x}(0) > 0$)

Eks: Størst komfort på humpete veier hvis støtdemperne gir $\gamma \approx \omega_0$.

Tvungen svingning og resonans [YF 14.8 ; LL 9.9]



$F_y(t) = F_0 \cos \omega t$ = ytre harmonisk kraft

N2: $-kx - b\dot{x} + F_0 \cos \omega t = m\ddot{x}$

$$\Rightarrow \ddot{x} + 2\gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \omega t \qquad \left(\gamma = \frac{b}{2m},\ \omega_0^2 = \frac{k}{m}\right)$$

Løsning: $x(t) = x_h(t) + x_p(t)$, der x_h er løsning av den homogene ligningen, som s. 59 og 60, dvs $x_h \approx 0$ etter en tid $t \gg 1/\gamma$, for da er $\exp(-\gamma t) \approx 0$.

Da er $x(t) = x_p(t)$, og vi gjetter

$$x_p(t) = A(\omega) \sin\left[\omega t + \varphi(\omega)\right],$$

hvoretter innsetting av x_p, $\dot{x}_p$ og $\ddot{x}_p$ i N2 gir

$$A(\omega) = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (2\gamma\omega)^2}} \quad ; \quad \varphi(\omega) = \arctan\left\{\frac{\omega_0^2 - \omega^2}{2\gamma\omega}\right\}$$

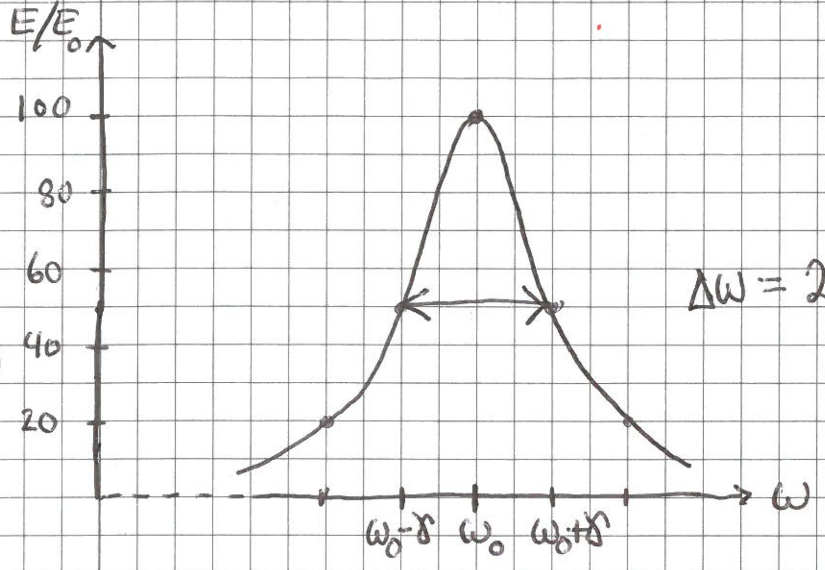
<u>Resonans</u> får vi med svak demping, $\gamma \ll \omega_0$, og $\omega \approx \omega_0$: Da kan $A(\omega)$ bli <u>svært stor</u>! Den ytre kraften driver systemet med frekvens ω lik systemets egenfrekvens (evt resonansfrekvens eller naturlige frekvens) $\omega_0 = \sqrt{k/m}$.

Hvis $\gamma \to 0$, vil $A(\omega) \to \infty$ når $\omega \to \omega_0$, jf. Tacoma bridge, 1940.

Oscillatorens energi:

$$E \approx \frac{1}{2}kA^2 = \frac{1}{2}k(F_0/m)^2 \frac{\omega_0^4/\omega_0^4}{(\omega^2-\omega_0^2)^2+(2\gamma\omega)^2}$$

$$= E_0 \cdot \frac{\omega_0^4}{(\omega^2-\omega_0^2)^2+(2\gamma\omega)^2} \quad ; \quad E_0 = F_0^2/2k$$



(Her: $\gamma = \omega_0/20$)

$\Delta\omega = 2\gamma$ = halvverdibredden

Oscillatorens Q-faktor ("quality"):

$$Q \equiv \omega_0/\Delta\omega = \omega_0/2\gamma \qquad (\text{Her: } Q=10)$$

Mål for hvor skarp resonanstoppen er.

Mindre demping $\Rightarrow$ smalere (og høyere) resonans

$\Rightarrow$ større Q-verdi

Exp 22.02.16:

Stav + 1 messinglodd $\Rightarrow T_0 \approx 0.65$ s, $f_0 \approx$ ~~1.5~~ 1.55 Hz (Målt)

Målt halvverdibredde: $\Delta f \approx 0.14$ Hz

$\Rightarrow Q = f_0/\Delta f \approx \underline{\underline{11}}$