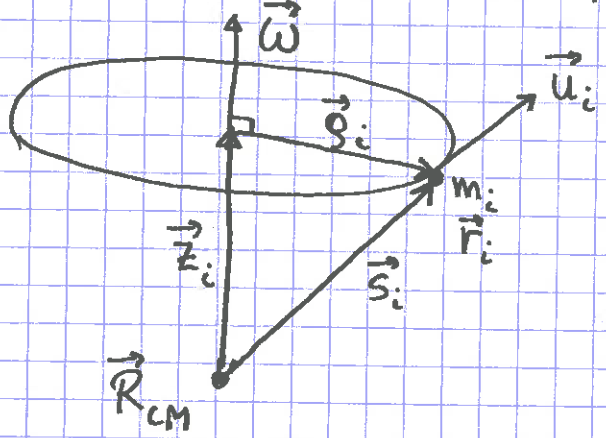
Bevis for $\vec{L} = M(\vec{R}_{CM} - \vec{r}_0) \times \vec{V} + I_0 \vec{\omega}$

for stivt legeme med ~~sylinder~~ refleksjons-symmetri om rotasjonsaksen



$$\vec{r}_i = \vec{R}_{CM} + \vec{s}_i$$
$$\vec{v}_i = \vec{V} + \vec{u}_i$$
$$\vec{s}_i = \vec{z}_i + \vec{\rho}_i$$

$$\vec{u}_i = \vec{\omega} \times \vec{\rho}_i = \vec{\omega} \times \vec{s}_i \qquad (\text{siden } \vec{\omega} \times \vec{z}_i = 0)$$

$$\begin{aligned}
\vec{L} &= \sum_i m_i (\vec{r}_i - \vec{r}_0) \times \vec{v}_i \\
&= \sum_i m_i (\vec{R}_{CM} - \vec{r}_0 + \vec{s}_i) \times (\vec{V} + \vec{u}_i) \\
&= \sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{V} + \sum_i m_i \vec{s}_i \times \vec{V} \\
&\quad + \sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{u}_i + \sum_i m_i \vec{s}_i \times \vec{u}_i
\end{aligned}$$

1. sum: $\sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{V} = M(\vec{R}_{CM} - \vec{r}_0) \times \vec{V} = \vec{L}_b$

2. sum: $\sum_i m_i \vec{s}_i \times \vec{V} = \sum_i m_i (\vec{r}_i - \vec{R}_{CM}) \times V = (M\vec{R}_{CM} - M\vec{R}_{CM}) \times \vec{V} = 0$

3. sum: $\sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times \vec{u}_i = \sum_i m_i (\vec{R}_{CM} - \vec{r}_0) \times (\vec{\omega} \times \vec{s}_i)$

$$= (\vec{R}_{CM} - \vec{r}_0) \times (\vec{\omega} \times \underbrace{\sum_i m_i \vec{s}_i}_{=0}) = 0$$

4. sum: $\sum_i m_i \vec{s}_i \times \vec{u}_i$ = dreieimpuls pga masseelementenes bevegelse relativt CM

(Til nå: <u>Helt</u> generelt!)

Stivt legeme: $\vec{u}_i = \vec{\omega} \times \vec{s}_i$

$$\Rightarrow \sum_i m_i \vec{s}_i \times \vec{u}_i = \sum_i m_i \vec{s}_i \times (\vec{\omega} \times \vec{s}_i)$$

Vektoridentitet: $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}\,(\vec{A}\cdot\vec{C}) - \vec{C}\,(\vec{A}\cdot\vec{B})$

$$\Rightarrow \sum_i m_i \vec{s}_i \times (\vec{\omega} \times \vec{s}_i) = \sum_i m_i \left\{ \vec{\omega}\, s_i^2 - \vec{s}_i\,(\vec{s}_i \cdot \vec{\omega}) \right\}$$

$$= \sum_i m_i \left\{ \vec{\omega}\,(z_i^2 + \rho_i^2) - (\vec{z}_i + \vec{\rho}_i)\, z_i \omega \right\}$$

$$= \sum_i m_i \left\{ \vec{\omega}\,(z_i^2 + \rho_i^2) - z_i^2 \vec{\omega} - z_i \omega \vec{\rho}_i \right\}$$

$$= \sum_i m_i \rho_i^2 \vec{\omega} - \omega \sum_i m_i z_i \vec{\rho}_i$$

$$= \underbrace{I_0 \vec{\omega}}_{\vec{L}_s} - \omega \sum_i m_i z_i \vec{\rho}_i$$

Med <u>~~speilsymmetri~~ refleksjons-symmetri om $\hat{\omega}$</u>, dvs om $\hat{z}$:

$$\sum_i m_i z_i \vec{\rho}_i = \sum_i m_i z_i (x_i \hat{x} + y_i \hat{y}) = 0$$

fordi bidraget fra (x_i, y_i) <u>kanselleres</u> av bidraget fra $(-x_i, -y_i)$!

Dermed:

$$\vec{L} = \vec{L}_b + \vec{L}_s = M(\vec{R}_{CM} - \vec{r}_0) \times \vec{V} + I_0 \vec{\omega}$$

qed