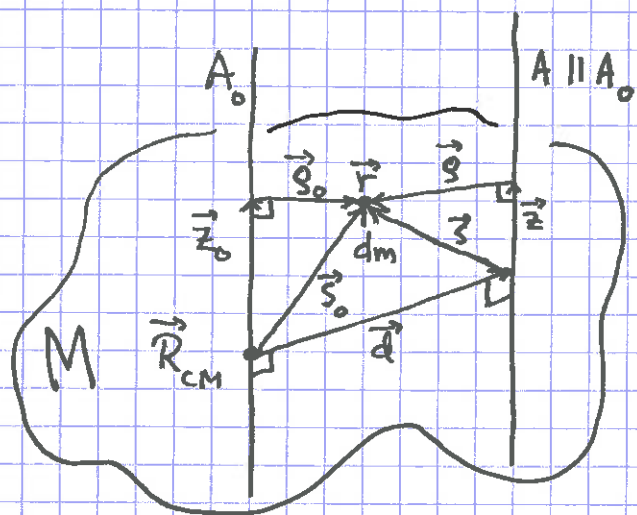


# Bevis Steiners sats



Fra figur: ( $dm$  er i pos.  $\vec{r}$ )

$$\vec{r} = \vec{R}_{CM} + \vec{s}_0$$

$$\vec{s}_0 = \vec{z}_0 + \vec{s}$$

$$\vec{s} = \vec{z} + \vec{s}_0$$

$$\vec{z} = \vec{z}_0 \quad (\text{fordi } \vec{d} \perp \hat{z})$$

$$\Rightarrow \vec{s} - \vec{s}_0 = \vec{z} - \vec{z}_0 = -\vec{d} \Rightarrow \vec{s} = \vec{s}_0 - \vec{d}$$

$$\Rightarrow s^2 = (\vec{s}_0 - \vec{d}) \cdot (\vec{s}_0 - \vec{d}) = s_0^2 + d^2 - 2\vec{d} \cdot \vec{s}_0$$

$$\Rightarrow I = \int s^2 dm = \underbrace{\int s_0^2 dm}_{I_0} + d^2 \cdot \underbrace{\int dm}_M - 2 \int \vec{d} \cdot \vec{s}_0 dm$$

$$\vec{d} \cdot \vec{s}_0 = \vec{d} \cdot (\vec{s}_0 - \vec{z}_0) = \vec{d} \cdot \vec{s}_0 = \vec{d} \cdot (\vec{r} - \vec{R}_{CM})$$

$$\Rightarrow \int \vec{d} \cdot \vec{s}_0 dm = \vec{d} \cdot \underbrace{\int \vec{r} dm}_{M\vec{R}_{CM}} - \vec{d} \cdot \vec{R}_{CM} \underbrace{\int dm}_M = 0$$

$$\Rightarrow I = I_0 + Md^2 \quad \text{qed}$$