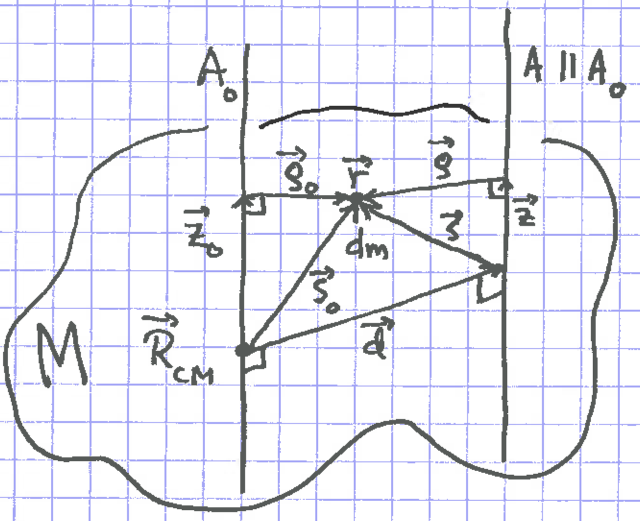
Bevis Steiners sats



Fra figur: (dm er i pos. $\vec{r}$)

$$\vec{r} = \vec{R}_{CM} + \vec{s}_0$$
$$\vec{s}_0 = \vec{z}_0 + \vec{\rho}_0$$
$$\vec{s} = \vec{z} + \vec{\rho}$$
$$\vec{z} = \vec{z}_0 \qquad (\text{fordi } \vec{d} \perp \hat{z})$$

$$\Rightarrow \vec{\rho} - \vec{\rho}_0 = \vec{s} - \vec{s}_0 = -\vec{d} \quad \Rightarrow \quad \vec{\rho} = \vec{\rho}_0 - \vec{d}$$

$$\Rightarrow \rho^2 = (\vec{\rho}_0 - \vec{d})\cdot(\vec{\rho}_0 - \vec{d}) = \rho_0^2 + d^2 - 2\vec{d}\cdot\vec{\rho}_0$$

$$\Rightarrow I = \int \rho^2\, dm = \underbrace{\int \rho_0^2\, dm}_{I_0} + d^2 \cdot \underbrace{\int dm}_{M} - 2\int \vec{d}\cdot\vec{\rho}_0\, dm$$

$$\vec{d}\cdot\vec{\rho}_0 = \vec{d}\cdot(\vec{s}_0 - z_0) = \vec{d}\cdot\vec{s}_0 = \vec{d}\cdot(\vec{r} - \vec{R}_{CM})$$

$$\Rightarrow \int \vec{d}\cdot\vec{\rho}_0\, dm = \vec{d}\cdot\underbrace{\int \vec{r}\, dm}_{M\vec{R}_{CM}} - \vec{d}\cdot\vec{R}_{CM}\underbrace{\int dm}_{M} = 0$$

$$\Rightarrow \quad I = I_0 + Md^2$$

qed