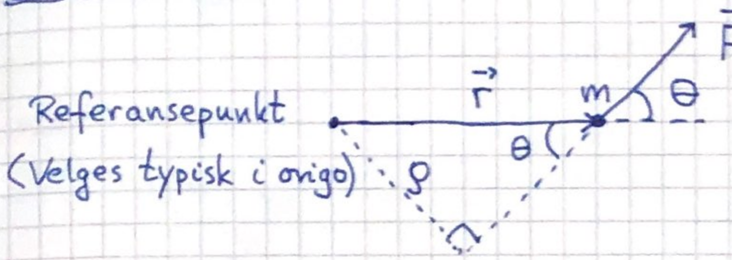
Rotasjonsdynamikk med vektorer

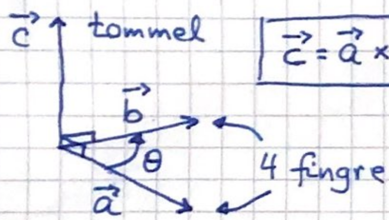
Dreiemoment : [OSI 10.6 ; YF 10.1 ; LL 5.5, 6.4]



$\vec{\tau} \overset{\text{def}}{=} \vec{r} \times \vec{F}$ = kraftens dreiemoment, relativt origo

Retning: $\vec{\tau} \perp \vec{r}$ og $\vec{\tau} \perp \vec{F}$

Høyrehåndsregel (HHR) for kryssprodukt :

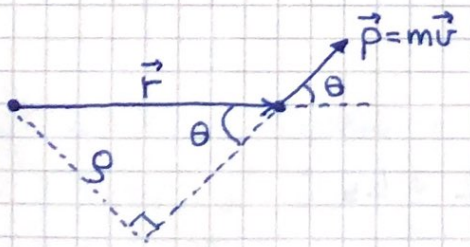


$\boxed{\vec{c} = \vec{a} \times \vec{b}}$ ⇒ $\vec{\tau}$ ut av planet ovenfor

Absoluttverdi : $\tau = r \cdot F \cdot \sin\theta = \rho \cdot F$; ρ = kraftens arm (som før)

Hvis flere ytre krefter: $\vec{\tau} = \sum_i \vec{\tau}_i = \sum_i \vec{r}_i \times \vec{F}_i$

Dreieimpuls : [OSI 11.2; YF 10.5 ; LL 6.6]



$\vec{L} \overset{\text{def}}{=} \vec{r} \times \vec{p}$

= massens dreieimpuls

$\vec{L} \perp \vec{r}$ og $\vec{L} \perp \vec{p}$

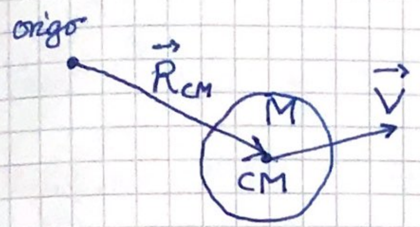
$L = r\,p\,\sin\theta = \rho \cdot p$

For partikkelsystem:

$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$ evt. $\vec{L} = \int \vec{r} \times \vec{v}\,dm$

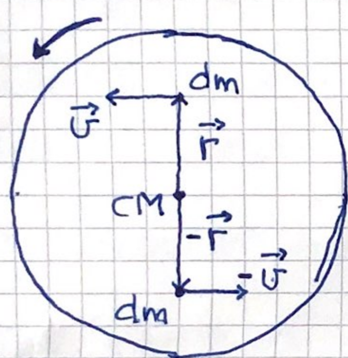
$\vec{L}$ for stivt legeme [OS1 11.2 ; YF 10.5 ; LL 6.6]

To bidrag: Banedreieimpuls $\vec{L}_b$ ("banespinn")
Indre dreieimpuls $\vec{L}_s$ ("spinn")



$$\vec{L}_b = \vec{R}_{CM} \times M\vec{V}$$

Anta legeme med refleksjonssymmetri om rotasjonsaksen, dvs uendret etter rotasjon $180°$, og anta $\vec{\omega} = \omega\hat{z}$.



Bidrag fra dm:

$$d\vec{L}_s = \vec{r} \times \vec{v}\, dm$$
$$= \varrho \cdot \varrho\omega\,\hat{z}\, dm = \varrho^2 dm \cdot \vec{\omega}$$
$$= dI_0 \cdot \vec{\omega}$$

(ϱ = avstand fra rot.aksen til dm)

$$\Rightarrow \vec{L}_s = \int d\vec{L}_s = \left\{ \int dI_0 \right\} \cdot \vec{\omega} = I_0 \vec{\omega}$$

Total dreieimpuls: $\vec{L} = \vec{L}_s + \vec{L}_b = I_0\vec{\omega} + \vec{R}_{CM} \times M\vec{V}$

[Se notat for mer detaljerte utledninger]

Eks: Jordas bevegelse relativt sola

$L_s = I_0\,\omega$; $I_0 \approx \frac{1}{3}MR^2$; $\omega = 2\pi/T_\omega$

$L_b = R_{CM}\,MV = R_{CM}^2\,M\,\Omega$; $\Omega = 2\pi/T_\Omega$

Tallverdier:

$M = 5.97\cdot10^{24}$ kg ; $R = 6.37\cdot10^{6}$ m ; $R_{CM} = 1.49\cdot10^{11}$ m

$T_\omega = 24$ h ; $T_\Omega = 365$ d ; $\angle(\vec{L}_s, \vec{L}_b) = 23.5°$

$\Rightarrow L_s = 5.9\cdot10^{33}$ Js ; $L_b = 2.6\cdot10^{40}$ Js $(\Rightarrow L \approx L_b)$

N2 for rotasjon [OSI 11.2 ; YF 10.5 ; LL 6.6]

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) \quad \text{siden} \quad \frac{d\vec{r}}{dt} \times \vec{p} = 0$$

$$\Rightarrow \boxed{\vec{\tau} = \frac{d\vec{L}}{dt}} \qquad \text{Jf.} \quad \vec{F} = \frac{d\vec{p}}{dt} \quad \text{for translasjon}$$

Hvis $\vec{F} = 0$ og $\vec{\tau} = 0$, er $\vec{p}$ og $\vec{L}$ konstante

Spesialtilfelle: Statisk likevekt, $\vec{p} = 0$ og $\vec{L} = 0$
[Jf. stigen s. 31]

Bevaringslover oppsummert

- Isolert system: Ingen ytre krefter.
  Da er total E, $\vec{p}$ og $\vec{L}$ bevart.
- Konservativt system: Mek. energi $K + U$ bevart.
- Netto ytre kraft $\vec{F} = 0 \Rightarrow \vec{p}$ bevart
- Netto ytre dreiemoment $\vec{\tau} = 0 \Rightarrow \vec{L}$ bevart

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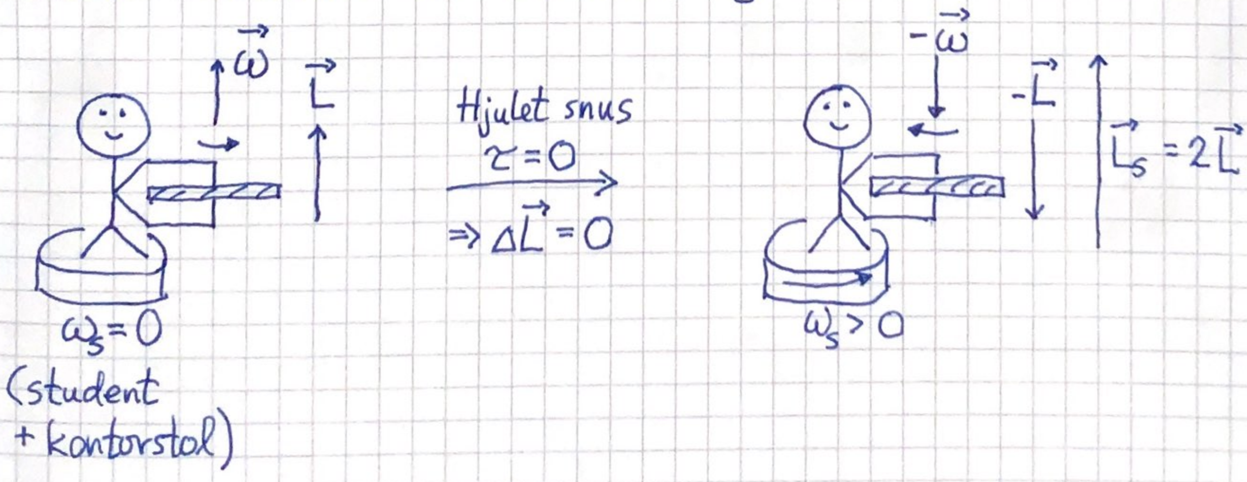
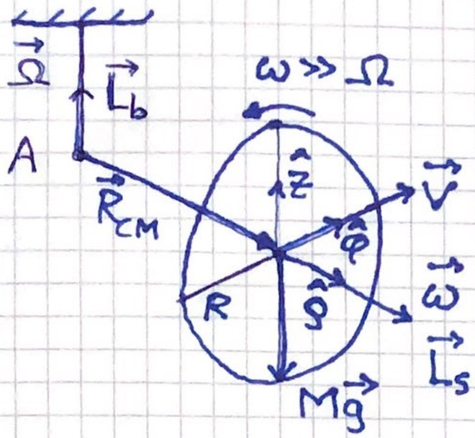
Eks 1: Student med roterende hjul



Eks 2: Presesjon [OSI 11.4; YF 10.7; LL 6.10]



$I_0 \approx MR^2$

$R \approx 0.3\,m$

$R_{CM} \approx 0.2\,m$

$T_\Omega = 2\pi/\Omega \approx 5\,s$

Beregn $T_\omega = 2\pi/\omega$

N2, rotasjon om A: $\vec{\tau}_A = d\vec{L}_A/dt$; $\vec{\tau}_A = \vec{R}_{CM} \times M\vec{g}$
$= R_{CM} M g\, \hat{\varphi}$

$\vec{L}_A = \vec{L}_s + \vec{L}_b$

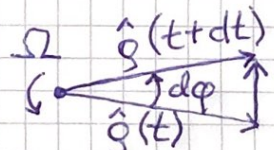
$\vec{L}_s = I_0 \vec{\omega} = MR^2 \omega\, \hat{\rho}$; $\vec{L}_b = \vec{R}_{CM} \times M\vec{V} = R_{CM}^2 M \Omega\, \hat{z}$

$\omega \gg \Omega \Rightarrow L_s \gg L_b \Rightarrow \vec{L}_A \approx \vec{L}_s$

Sett inn i N2: $R_{CM} M g\, \hat{\varphi} = MR^2 \omega\, d\hat{\rho}/dt$

Akslingen og $\hat{\rho}$ roterer om z-aksen med vinkelfart Ω

$\Rightarrow d\hat{\rho}/dt = \Omega \cdot \hat{\varphi}$



$d\hat{\rho} = 1 \cdot d\varphi \cdot \hat{\varphi}$
$= \Omega\, dt\, \hat{\varphi}$

Dermed:

$$R_{CM} M g = MR^2 \omega \Omega = MR^2 \frac{2\pi}{T_\omega} \frac{2\pi}{T_\Omega}$$

$$\Rightarrow T_\omega = \frac{(2\pi R)^2}{R_{CM}\, g\, T_\Omega} \approx \frac{(2\pi \cdot 0.3)^2}{0.2 \cdot 10 \cdot 5}\, s \approx \underline{\underline{0.4\,s}}$$

dvs $f_\omega = 1/T_\omega \approx 2.5$ omdreininger pr sekund