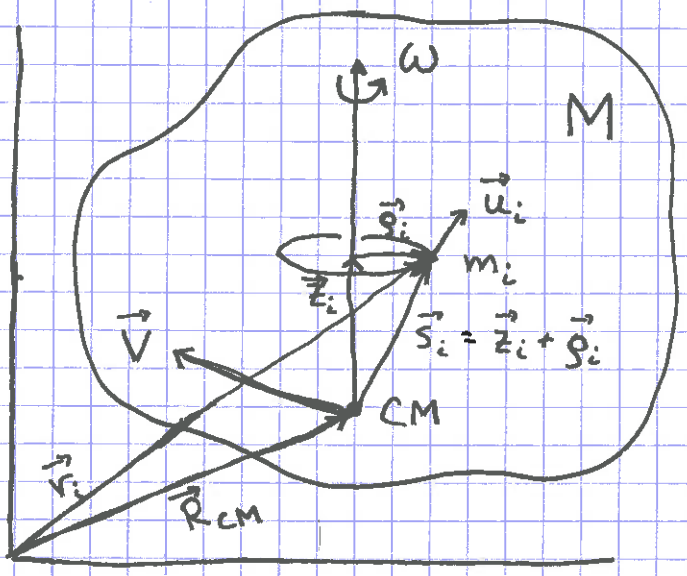
Bevis for $K = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2$ for stivt legeme



$$\vec{r}_i = \vec{R}_{CM} + \vec{s}_i$$

$$\vec{v}_i = \dot{\vec{r}}_i = \dot{\vec{R}}_{CM} + \dot{\vec{s}}_i = \vec{V} + \vec{u}_i$$

$$v_i^2 = V^2 + u_i^2 + 2\vec{V}\cdot\vec{u}_i$$

$$u_i^2 = (\varrho_i\omega)^2$$

$$K = \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}\underbrace{\sum_i m_i}_{M} V^2 + \frac{1}{2}\sum_i m_i u_i^2 + \vec{V}\cdot\sum_i m_i\vec{u}_i$$

$$= \frac{1}{2}MV^2 + \frac{1}{2}\underbrace{\left\{\sum_i m_i\varrho_i^2\right\}}_{I_0}\omega^2 + \vec{V}\cdot\frac{d}{dt}\sum_i m_i\vec{s}_i$$

$$\sum_i m_i\vec{s}_i = \sum_i m_i(\vec{r}_i - \vec{R}_{CM}) = M\vec{R}_{CM} - M\vec{R}_{CM} = 0$$

$$\Rightarrow \quad K = \frac{1}{2}MV^2 + \frac{1}{2}I_0\omega^2$$

qed