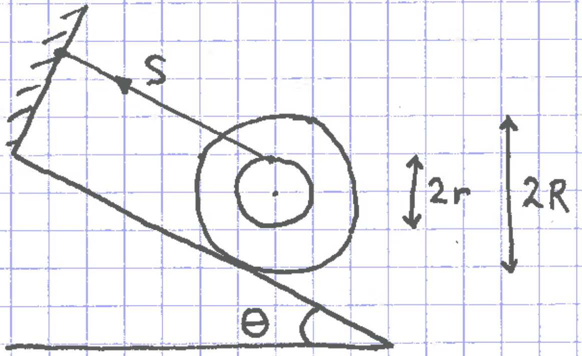
Eks 3: Sluresnelle



Finn max vinkel θ_0 uten at snella glir nedover skråplanet. Hva er S da?

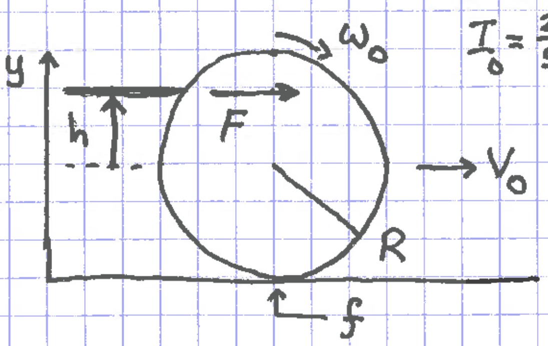
Tips: N1, ∥ ; N1, rot. om CM ; $f = f_{max} = \mu_s N$ når $\theta = \theta_0$

Finn også snellas akselerasjon $a = \dot{v} = r\dot{\omega}$ når $\theta > \theta_0$

Tips: N2, ∥ ; N2, rot. om CM ; $f = \mu_k N$

$\boxed{29.09.14}$

Eks 4: Snooker [LL 6.7]



$I_0 = \frac{2}{5} MR^2$

Kort støt:

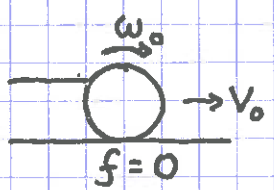
$F \cdot \Delta t = \Delta p = MV_0$ (N2)

$$\left.\begin{aligned} \tau \cdot \Delta t &= \Delta L = I_0 \omega_0 \\ \tau &= F \cdot h \end{aligned}\right\} \text{(N2, rot. om CM)}$$

($F \gg f$ i støtet)

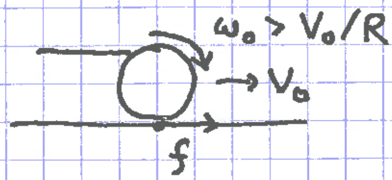
Etter støtet:

Ren rulling hvis $h = h_0$, $V_0 = \omega_0 R$:



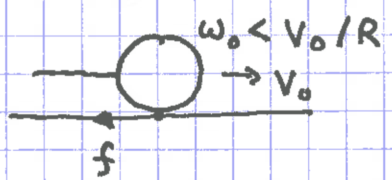
$h_0 = ?$

$h > h_0$:



Sluring

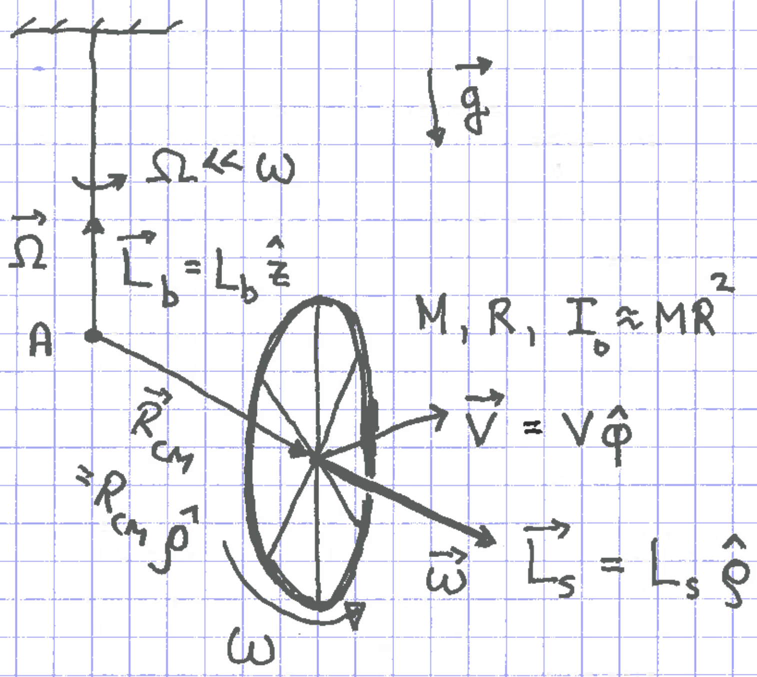
$h < h_0$:



Sluring

} Etter hvert ren rulling

Eks 5: Presesjon [YF 10.7 ; LL 6.10]



Tallverdier, sykkelhjul:

$M \approx 5$ kg

$R = 0{,}3$ m

$R_{CM} = 0{,}2$ m

Målt på forelesn:

$T_\Omega = \frac{2\pi}{\Omega} = 4.4$ s

Oppgave: Beregn $T_\omega = 2\pi/\omega$.

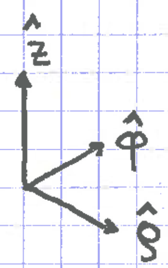
Løsning:

A = valgt ref. punkt ; $\vec{r}_0 = \vec{r}_A = 0$

N2, rot. mhp A : $\vec{\tau}_A = d\vec{L}_A/dt$

Tyngden $M\vec{g}$ har armen $\vec{R}_{CM}$ mhp A

$\Rightarrow \vec{\tau}_A = \vec{R}_{CM} \times M\vec{g} = R_{CM} Mg \ \hat{\varphi}$



(Snordraget $\vec{S}$ har ingen arm mhp A.
N1 vertikalt: $\vec{S} + M\vec{g} = 0$; $|\vec{S}| = Mg$)

Hjulets dreieimpuls (mhp A):

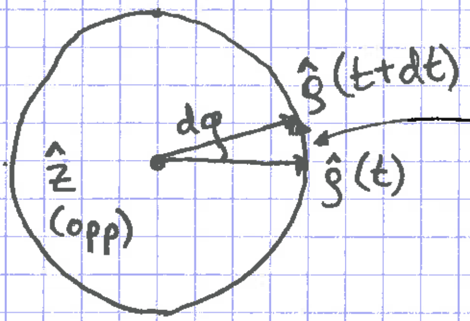
$$\vec{L}_A = \vec{L}_b + \vec{L}_s = \vec{R}_{CM} \times M\vec{V} + I_0\vec{\omega}$$

$$= R_{CM} MV \hat{z} + MR^2\omega \hat{\rho}$$

$\Rightarrow \vec{L}_b$ er konstant (neglisjerer friksjon)

$$\Rightarrow \frac{d\vec{L}_A}{dt} = \frac{d\vec{L}_s}{dt} = MR^2 \frac{d\vec{\omega}}{dt} = MR^2 \omega \frac{d\hat{\rho}}{dt}$$

$\Rightarrow d\vec{\omega}$ (og $d\hat{\rho}$) <u>må</u> peke i retning $\hat{\varphi}$, dvs som $\vec{\tau}$:



$$d\hat{\rho} = d\varphi \cdot |\hat{\rho}| \cdot \hat{\varphi} = d\varphi \cdot \hat{\varphi}$$

$$\Rightarrow \dot{\vec{L}}_A = MR^2 \omega \frac{d\varphi}{dt} \hat{\varphi} = MR^2 \omega \Omega \hat{\varphi}$$

$$\Rightarrow \underbrace{R_{CM} Mg}_{\tau} = \underbrace{MR^2 \omega \Omega}_{\dot{L}_A} \Rightarrow \omega = R_{CM}\, g / R^2 \Omega$$

$$\Rightarrow T_\omega = \frac{2\pi}{\omega} = \frac{(2\pi R)^2}{R_{CM}\, g\, T_\Omega}$$

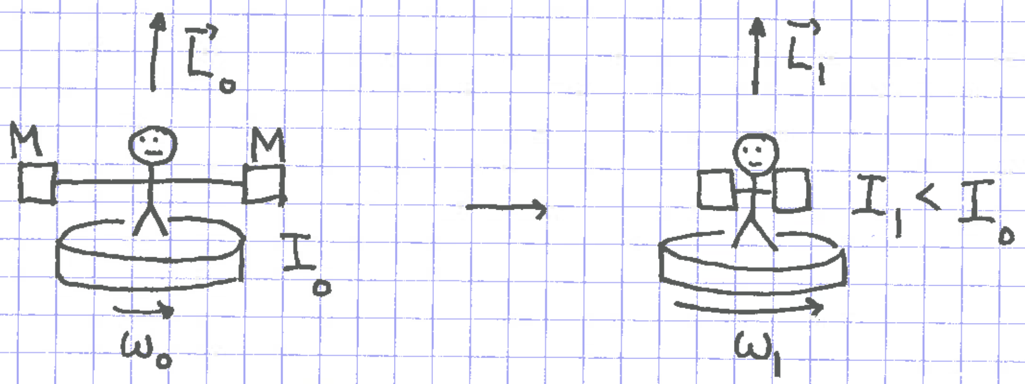
Med tallverdier: $(g \approx 10\ \mathrm{m/s^2})$

$$T_\omega = \frac{(2\pi \cdot 0.3)^2}{0.2 \cdot 10 \cdot 4.4} = \underline{0.40\ \mathrm{s}}$$

Rimelig: ca $2\frac{1}{2}$ omdreining pr sekund.

Eks 6: Piruett

[YF 10.6; LL 6.5]

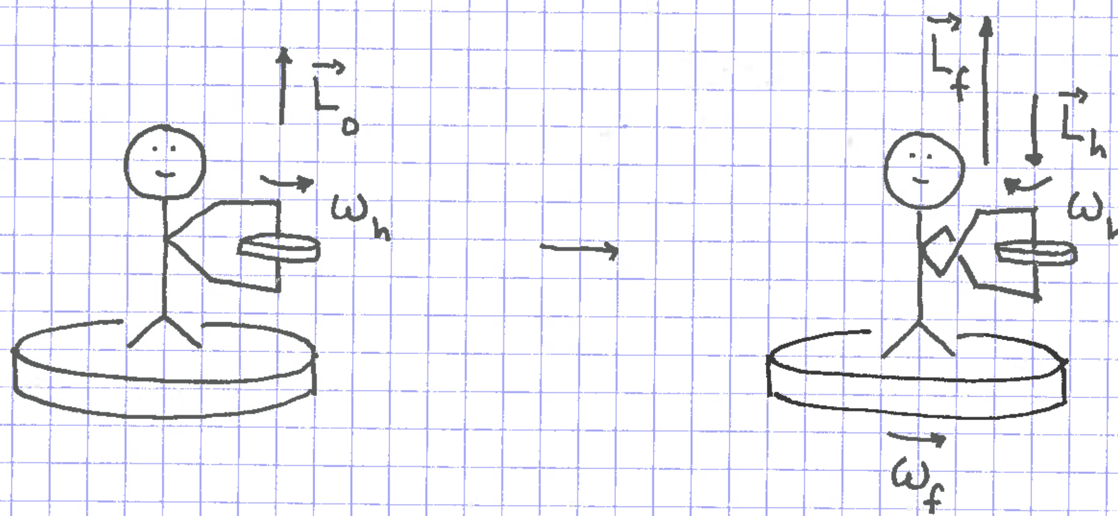


$$\tau_{ytre} = 0 \Rightarrow \vec{L}_0 = \vec{L}_1 \Rightarrow I_0\omega_0 = I_1\omega_1 \Rightarrow \omega_1 = \frac{I_0}{I_1}\omega_0 > \omega_0$$

[Mek. energi øker: $K_1 = \frac{1}{2}I_1\omega_1^2 > \frac{1}{2}I_0\omega_0^2 = K_0$.

Vis dette! Forklar hvorfor!]

Eks 7: Dreieimpulsbevarelse for hjul + foreleser (+ stol)

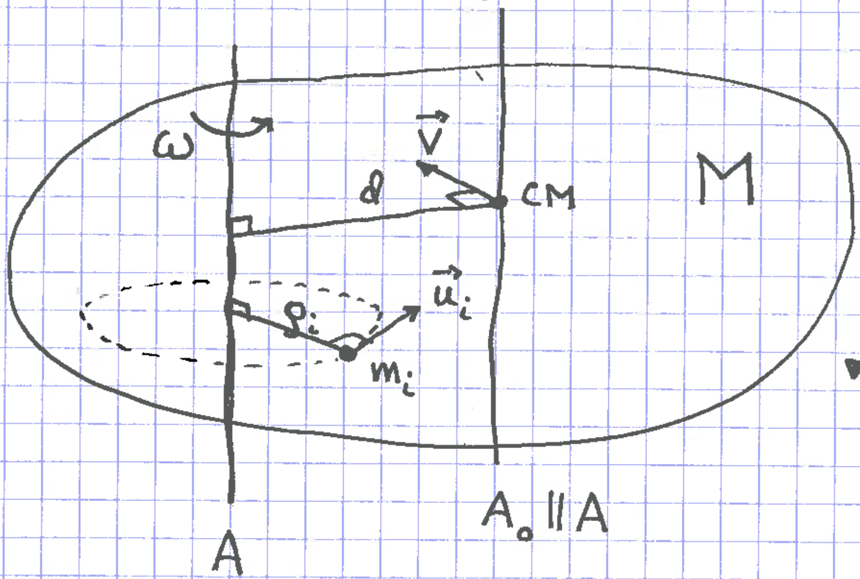


$$\tau_{ytre} = 0 \Rightarrow \vec{L}_f + \vec{L}_h = \vec{L}_0$$

$$\Rightarrow \vec{L}_f = \vec{L}_0 - \vec{L}_h = 2\vec{L}_0$$

Rotasjon av stivt legeme om akse med fast orientering

[YF 10.5 ; LL 6.2, 6.4]



Rot. om fast akse A

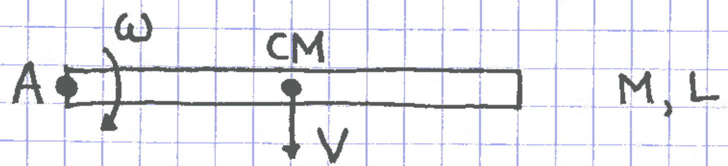
$$K = \frac{1}{2}\sum_i m_i u_i^2 = \frac{1}{2}\left\{\sum_i m_i \rho_i^2\right\}\omega^2 = \underline{\frac{1}{2} I \omega^2}$$

Steiners sats : $I = I_0 + Md^2$

$$\Rightarrow K = \frac{1}{2} I_0 \omega^2 + \frac{1}{2} M d^2 \omega^2 \overset{V = \omega \cdot d}{=} \underline{\frac{1}{2} I_0 \omega^2 + \frac{1}{2} M V^2}$$

(som utledet generelt; se s. 40)

Eks :



Tynn stav, masse M, lengde L, rot. om A.

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{1}{3} ML^2 \cdot \omega^2 = \frac{1}{6} ML^2 \omega^2$$

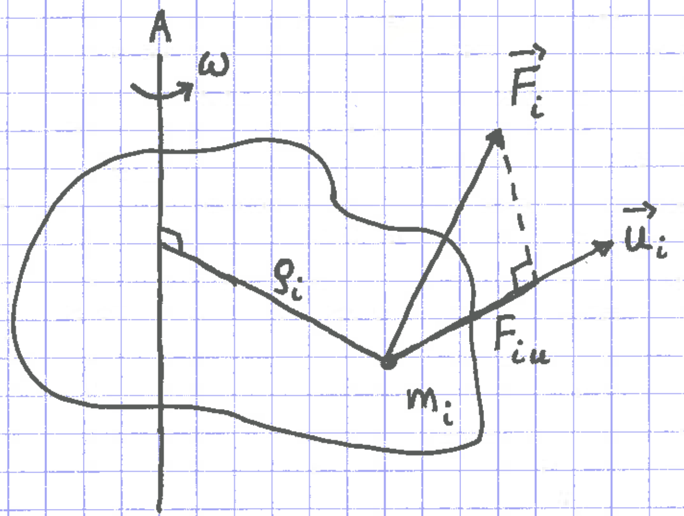
evt. $(V = \omega L/2)$

$$K = \frac{1}{2} I_0 \omega^2 + \frac{1}{2} M V^2 = \frac{1}{2} \cdot \frac{1}{12} ML^2 \omega^2 + \frac{1}{2} M (\omega L/2)^2$$

$$= \left(\frac{1}{24} + \frac{1}{8}\right) ML^2 \omega^2 = \frac{1}{6} ML^2 \omega^2$$

Samme svar !

N2, rot. om fast akse:



Tilført effekt:

$$P = \sum_i \vec{F}_i \cdot \vec{u}_i \overset{N2}{=} \sum_i m_i \frac{d\vec{u}_i}{dt} \cdot \vec{u}_i \overset{\text{som s.21}}{=} \frac{d}{dt} \sum_i \frac{1}{2} m_i u_i^2 \quad \left(= \frac{dK}{dt}\right)$$

$$= \frac{d}{dt} \left\{ \sum_i m_i \rho_i^2 \right\} \cdot \frac{1}{2} \omega^2 = I \cdot \frac{1}{2} \cdot 2\omega \cdot \frac{d\omega}{dt}$$

Dessuten er:

$$\sum_i \vec{F}_i \cdot \vec{u}_i = \sum_i F_{iu} u_i = \left\{ \sum_i F_{iu} \rho_i \right\} \omega = \tau \cdot \omega$$

Dermed:

$$\boxed{\tau = I \dot{\omega}}$$

Merk at dreiemomentet $\tau = \sum_i F_{iu} \rho_i$ her refererer til rotasjonsaksen, og ikke til et bestemt punkt ($\vec{r}_0$) på rot. aksen. Da gjelder alltid $\tau = I\dot{\omega}$.

Gjelder også med fast orientering av rot. aksen (som f.eks. ved ~~[illegible]~~ rulling nedover skråplanet, s. 57)

SVINGNINGER [YF 14; LL 9]

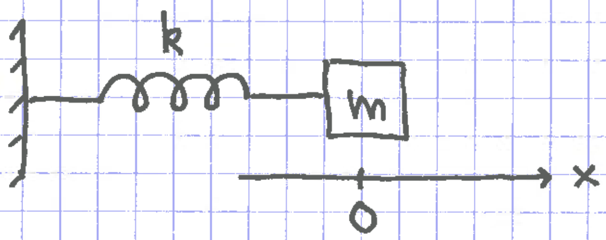
02.10.14

= oscillasjoner

= periodisk oppførsel omkring likevekt

Eks: masse/fjær, pendel, gitarstreng, luft i orgelpipe, atomer i molekyler og krystaller ...

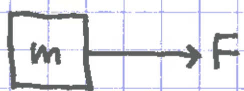
Harmonisk oscillator [YF 14.2; LL 9.1-9.3]



Likevekt ($F = 0$) med m (CM) i $x = 0$

Strukket fjær, $x > 0$:

Sammenpresset fjær, $x < 0$:

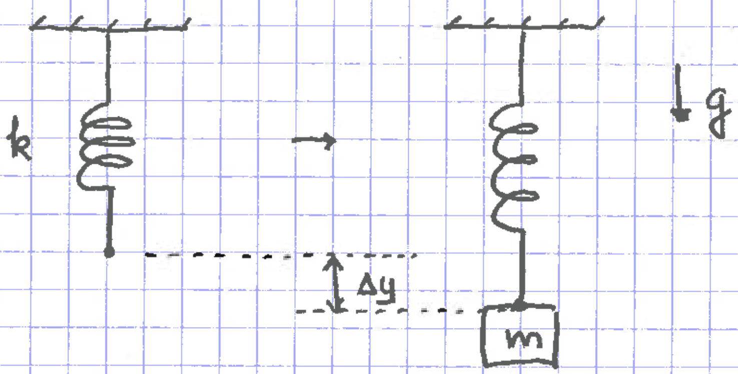


Hookes lov (ideell fjær): $|F| \sim |x|$

$$\Rightarrow \boxed{\vec{F} = -kx\,\hat{x}}$$

Fjærkonstanten: k $\qquad [k] = \mathrm{N/m}$

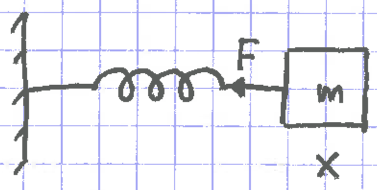
Vertikalt i tyngdefeltet:



Likevekt:

$k\,\Delta y = mg$

$\Rightarrow \quad \Delta y = mg/k$



N2: $\quad -kx = m\ddot{x}$

$\Rightarrow \quad \ddot{x} + \frac{k}{m}x = 0$

Innfør $\omega_0 = \sqrt{k/m} \quad \Rightarrow \quad \boxed{\ddot{x} + \omega_0^2 x = 0}$ Enkel harm. osc. i en dimensjon

Løsning: Frie svingninger uten demping.

Ser at både $\sin\omega_0 t$ og $\cos\omega_0 t$ løser ligningen, siden $\frac{d^2}{dt^2}(\sin\omega_0 t) = -\omega_0^2 \sin\omega_0 t$, og tilsvarende for $\cos\omega_0 t$.

Generell løsning:

$x(t) = A\cos(\omega_0 t + \varphi)$ evt $x(t) = B\cos\omega_0 t + C\sin\omega_0 t$

Int. konst. A, φ evt. B, C fastlegges med 2 initialbetingelser, f.eks: $x(0) = x_0$, $\dot{x}(0) = v_0$.

[$\cos(a+b) = \cos a\cos b - \sin a \sin b$ gir relasjoner mellom A, φ og B, C]

$x(t) = A\cos(\omega_0 t + \varphi)$

A = amplitude = max utsving fra likevekt ; $[A] = [x]$

ω_0 = vinkelfrekvens ; $[\omega_0] = s^{-1}$

$T = 2\pi/\omega_0$ = periode = tid pr svingning ; $[T] = s$

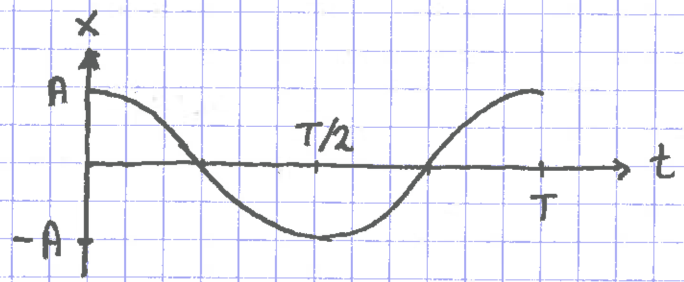
$f = 1/T$ = frekvens = svingninger pr tidsenhet ; $[f] = Hz = s^{-1}$

$\omega_0 t + \varphi$ = svingningens fase

φ = fasekonstant ; $[\varphi] = 1$

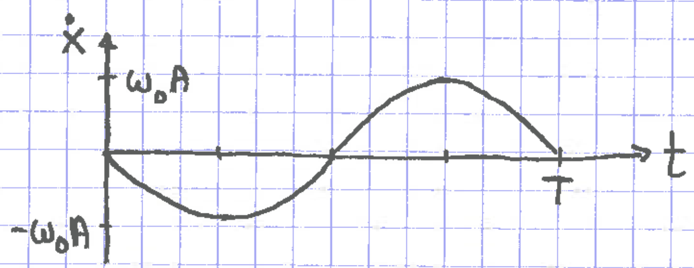
Anta f.eks. $\varphi = 0$ og $A > 0$:

$x(t) = A\cos\omega_0 t$



$\dot{x}(t) = -\omega_0 A \sin\omega_0 t$

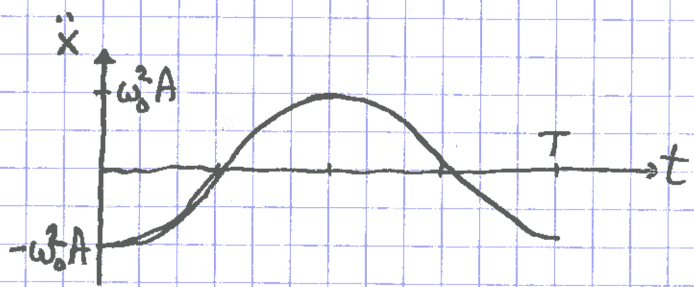
$= \omega_0 A \cos(\omega_0 t + \frac{\pi}{2})$



$\ddot{x}(t) = -\omega_0^2 A\cos\omega_0 t$

$= -\omega_0^2 x$

$= \omega_0^2 A\cos(\omega_0 t + \pi)$



Energi i harmonisk oscillator [YF 14.3; LL 9.4]

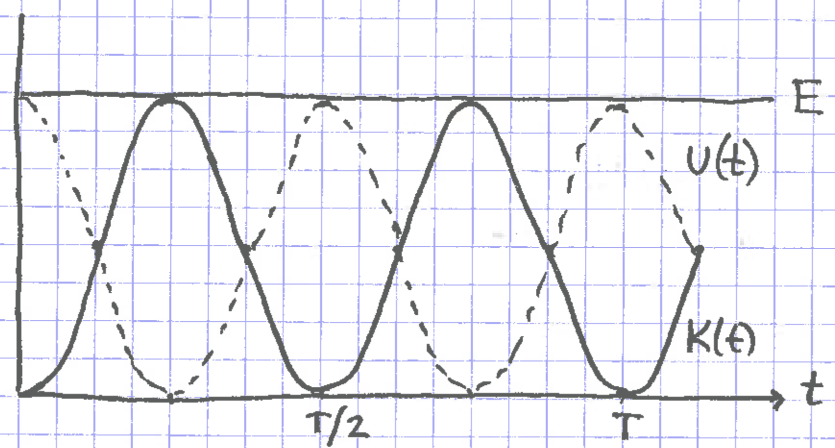
$$K(t) = \frac{1}{2} m \dot{x}(t)^2 = \frac{1}{2} m \omega_0^2 A^2 \sin^2 \omega_0 t = \frac{1}{2} k A^2 \sin^2 \omega_0 t$$

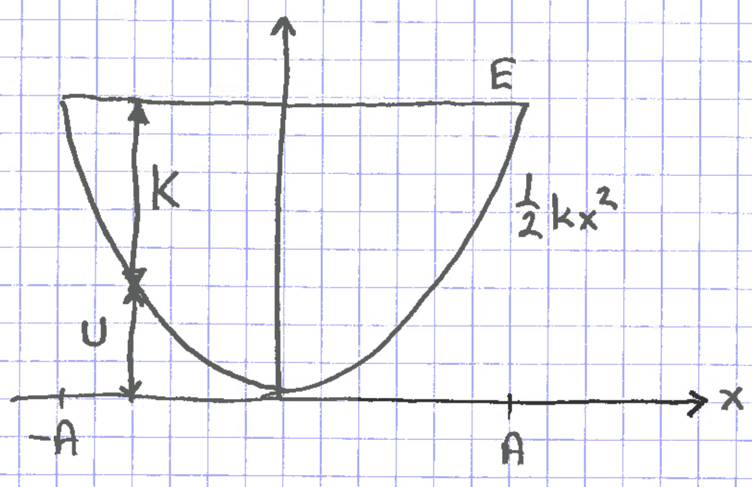
$$U = -\int_0^x F(x)\,dx = -\int_0^x (-kx)\,dx = \frac{1}{2} k x^2$$

$$U(t) = \frac{1}{2} k x(t)^2 = \frac{1}{2} k A^2 \cos^2 \omega_0 t$$

$\Rightarrow$ Systemet er konservativt, dvs E er bevart (E = mek. energi):

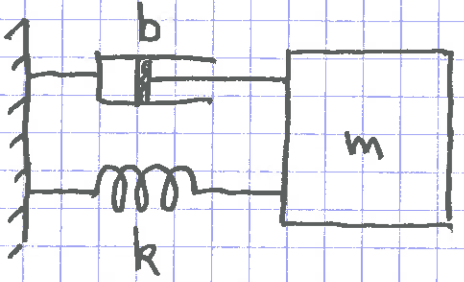
$$E = K + U = \frac{1}{2} k A^2 (\sin^2 \omega_0 t + \cos^2 \omega_0 t) = \frac{1}{2} k A^2 = \text{konst.}$$





Dempet fri svingning [YF 14.7 ; LL 9.7]

Antar $f = -b\dot{x}$, dvs friksjon i fluid (langsom bevegelse).



Netto kraft på m:

$-kx - b\dot{x}$

N2 : $-kx - b\dot{x} = m\ddot{x}$

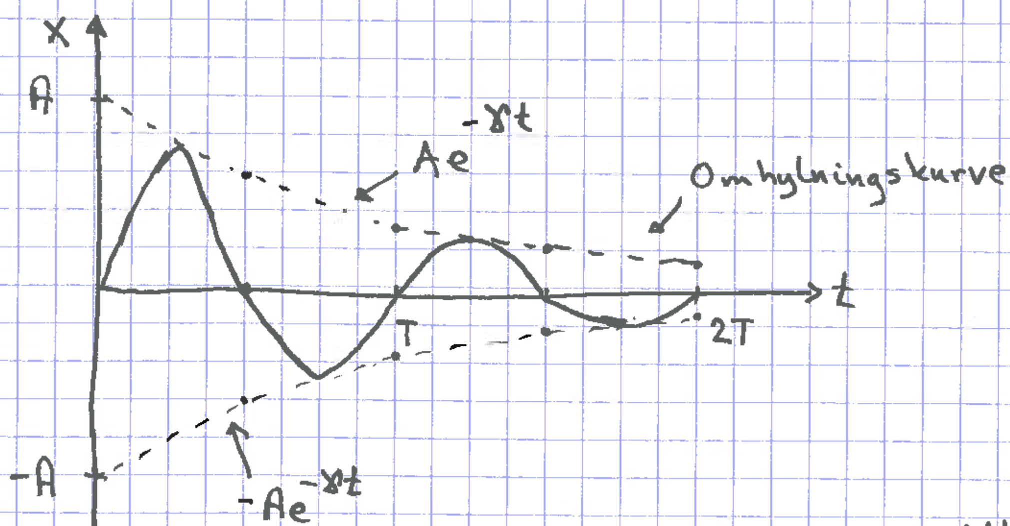
$\Rightarrow \ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$; $\gamma = b/2m$, $\omega_0^2 = k/m$

$[\gamma] = [\omega_0] = s^{-1}$

Løsning:

Underkritisk (svak) demping, $\gamma < \omega_0$ $(b < 2\sqrt{k \cdot m})$

$x(t) = Ae^{-\gamma t}\sin(\omega t + \varphi)$; $\omega = \sqrt{\omega_0^2 - \gamma^2} < \omega_0$

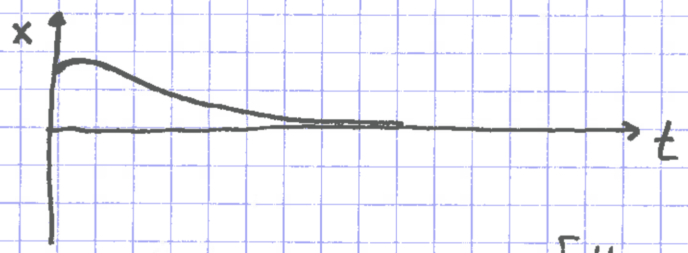


$\Rightarrow$ Amplituden $Ae^{-\gamma t}$ avtar eksponentiell med t

Overkritisk demping, $\gamma > \omega_0$:

$$x(t) = A e^{-\alpha_1 t} + B e^{-\alpha_2 t}$$

$$\alpha_1 = \gamma + \sqrt{\gamma^2 - \omega_0^2}, \quad \alpha_2 = \gamma - \sqrt{\gamma^2 - \omega_0^2}$$

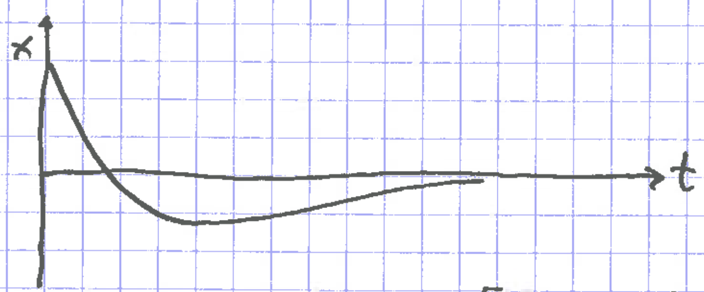


Ingen svingninger

[Her: $x(0) > 0$, $\dot{x}(0) > 0$]

Kritisk demping, $\gamma = \omega_0$:

$$x(t) = A e^{-\gamma t} + B t e^{-\gamma t}$$



[Her: $x(0) > 0$, $\dot{x}(0) < 0$]

Eks: Støtdempere i (f.eks.) bil.

Mest behagelig med $\gamma \approx \omega_0$ på humpete veier