Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Spring 2005

Øving 4

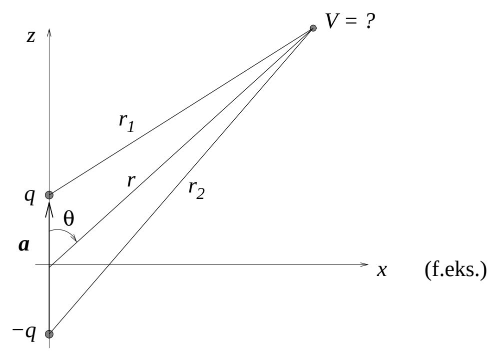
Guidance: February 3. and 4.
Deliver no later than: Monday February 7

Exercise 1

Assume we have a uniform electric field $\boldsymbol{E} = E_0\,\hat{x}$. Determine the potential difference between the origin and the following points (x, y) in the xy plane:
(i) $(a, 0)$ (ii) $(0, a)$ (iii) the point (a, a)

Exercise 2

An *electric dipole* is located along the z-axis with the centre at the origin, as shown in the figure. The electric *dipole moment* is defined as $\boldsymbol{p} = q\boldsymbol{a}$, where $\boldsymbol{a} = a\,\hat{z}$ is the vector from $-q$ to q.



Since we here obviously must have *symmetry* with respect to a rotation around the z-axis, it is sufficient to investigate a plane containing the z-axis. We have here chosen the xz plane. Further, we may choose between cartesian coordinates (x, z) or polar coordinates (r, θ) in order to denote an arbitrary position in this plane. We will use both in this exercise. The polar angle θ may be chosen with respect to any one of the cartesian axes. Here, we let θ denote the angle between $\boldsymbol{r}$ and the z axis (see figure).

a) First, write down the relation between the cartesian coordinates (x, z) and the polar coordinates (r, θ). I.e., determine $x(r, \theta)$, $z(r, \theta)$ and $r(x, z)$.

b) Show that the potential from such a dipole, in cartesian coordinates, becomes

$$V(x, z) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2 + (z - a/2)^2}} - \frac{1}{\sqrt{x^2 + (z + a/2)^2}}\right)$$

What is the potential on the x axis, $V(x, 0)$? What is the potential on the z axis, $V(0, z)$? (I.e., on the *complete* z axis; be careful with the signs...!) Draw a sketch of the function $V(0, z)$.

c) Show that far away from the dipole (i.e., $r \gg a$), the potential is to a good approximation given (in polar coordinates) by

$$V(r,\theta) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\varepsilon_0 r^3}$$

Hint: Start with

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{r_2 - r_1}{r_1 r_2}$$

and use the figure to find an approximate expression for this when $r \gg a$.
Whereas the potential from a single point charge goes to zero as $1/r$, the potential from a dipole goes *faster* to zero, namely as $1/r^2$. Is this result reasonable?

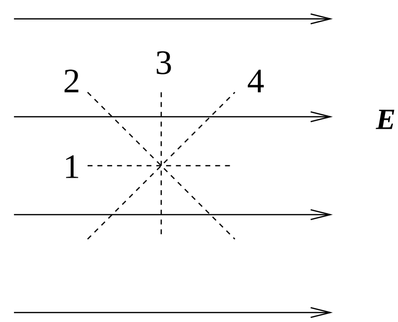
Comment: If you insist on a more rigid mathematical approach to things like this, we are talking about finding $V(r,\theta)$ "to leading order" in the small parameter a/r. In other words, the given expression for $V(r,\theta)$ is *exact* for a socalled *ideal dipole* with "zero extent" (i.e., in the limit $a \to 0$, keeping the value of p constant). Extra, if you think all this was too easy: What is the "dominating correction" to the given $V(r,\theta)$? I.e., what is the next term in the series expansion (Maclaurin series) of $V(r,\theta)$ for small values of a/r, i.e., $a/r \ll 1$?

Exercise 3 (from earliger midterm exams)

a) The figure shows field lines for a uniform electric field. An electron which is placed in this field will

A move with constant speed to the left.
B move with constant speed to the right.
C be accelerated to the left.
D be accelerated to the right.

b) The figure shows a uniform electric field $\boldsymbol{E}$ (solid lines). Along which stapled line does the potential remain unchanged?

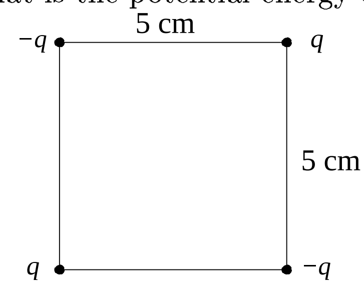


A 1
B 2
C 3
D 4

c) A particle with negative charge is placed with zero initial velocity in an electrostatic field $\boldsymbol{E}$. The movement of the particle will be

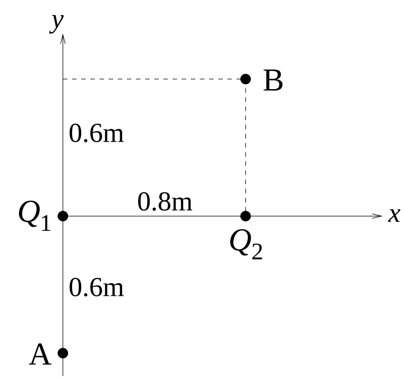
A in the direction of lower potential.
B in the direction of lower potential energy.
C in the same direction as $\boldsymbol{E}$.
D in a direction perpendicular to $\boldsymbol{E}$.

d) Four point charges, two positive and two negative ($q = 9\ \mu$C), are located in the corners of a square with edges 5 cm, as shown in the figure. What is the potential energy of the system?



A 19 J
B Zero
C -7 J
D -38 J

e) Two point charges $Q_1 = 69$ nC and $Q_2 = -98$ nC are located in the xy plane, as shown in the figure. An electron is moved from point A to point B. How big is the change in the potential energy of the system, as a result of this movement? (The "system" = the two point charges and the electron.) (1 eV $= 1.6 \cdot 10^{-19}$ J)



A -1 keV
B -1 eV
C 1 eV
D 1 keV

f) The potential energy of two electrons with a mutual distance 1 Å ($= 10^{-10}$ m) is [1 eV $= 1.6 \cdot 10^{-19}$ J]

A 14.4 meV
B 14.4 eV
C 14.4 keV
D 14.4 MeV

g) A beryllium nucleus with charge $4e$ and mass $9m_p$ and an α particle (i.e., a helium nucleus) with charge $2e$ and mass $4m_p$ are both at rest. The two particles can be brought to equal speed by

A accelerating them through an equal potential difference.
B accelerating the α particle through V volts and the beryllium nucleus through $V/2$ volts.
C accelerating the α particle through V volts and the beryllium nucleus through $8V/9$ volts.
D accelerating the α particle through V volts and the beryllium nucleus through $9V/8$ volts.