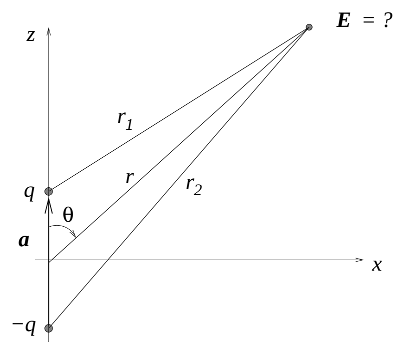
Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2005

Øving 5

Guidance: February 10 and 11
To be delivered by: Monday February 14

Exercise 1



In exercise 2 in øving 4, we investigated an electric dipole, consisting of two point charges $\pm q$ located on the z axis in $z = \pm a/2$. We showed that the potential V far away ($r \gg a$) from the dipole is approximately equal to

$$V(r, \theta) = \frac{p \cos \theta}{4\pi\varepsilon_0 r^2}$$

Here, r is the distance form the origin, i.e., the centre of the dipole, θ is the angle between the z axis and $\boldsymbol{r}$, and $p = |\boldsymbol{p}| = qa$ is the electric dipole moment of the dipole.

a) Starting from the expression above for $V(r, \theta)$, determine the electric field $\boldsymbol{E}(r, \theta) = E_r \hat{r} + E_\theta \hat{\theta}$ far away from the dipole.
The gradient operator in spherical coordinates is

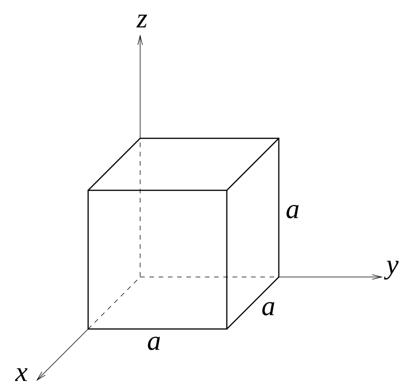
$$\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r \sin \theta}\frac{\partial}{\partial \phi}$$

I will not provide the answer, but you can to some extent control your answer by checking that the result is reasonable for $\theta = 0$ and for $\theta = \pi/2$. What about $r = 0$?

b) Because of rotational symmetry around the z axis, we may e.g. assume that we are in the xz plane. Determine the electric field $\boldsymbol{E}(x, z) = E_x \hat{x} + E_z \hat{z}$ in cartesian coordinates for $r \gg a$. Hint: Start with the expressions you found for E_r amd E_θ in a). Make a figure and find the relation between the coordinates (x, z) and (r, θ), and the components of the electric field, E_x, E_z and E_r, E_θ.
[Answer: $E_x = 3pxz/4\pi\varepsilon_0(x^2 + z^2)^{5/2}$, $E_z = p(2z^2 - x^2)/4\pi\varepsilon_0(x^2 + z^2)^{5/2}$.]

c) Also find $\boldsymbol{E}(x,z)$ by first rewriting $V(r,\theta)$ in cartesian coordinates, and then using the gradient operator in cartesian coordinates on $V(x,z)$.

Exercise 2



The figure above shows a gaussian surface (i.e., a closed surface) S formed as a cube with sides a. The surface is located in a region where there exists an electric field $\boldsymbol{E}$. In each of the cases *a*) – *d*) below, determine the total (net) electric flux ϕ that passes through the surface S. Then use Gauss' law and find in each case also the total charge Q inside S.

$$\begin{aligned} a)\ \boldsymbol{E} &= C\hat{x} \\ b)\ \boldsymbol{E} &= Cx\hat{x} \\ c)\ \boldsymbol{E} &= Cx^2\hat{x} \\ d)\ \boldsymbol{E} &= C\left(y\hat{x} + x\hat{y}\right) \end{aligned}$$

Here, C is a (scalar) constant (with different units in the different cases, of course).

e) For *c*) above, determine the charge density (i.e., charge pr unit volume) ρ inside S. Hint: Use Gauss' law with a gaussian surface enclosing a thin slice with thickness dx and top and bottom surfaces with area a^2, located between x and $x + dx$. (So the volume of the slice is $a^2\ dx$.)

Some answers: *b*): $Q = C\varepsilon_0 a^3$ *c*): $Q = C\varepsilon_0 a^4$ *e*): $\rho = 2C\varepsilon_0 x$

Exercise 3

Use Gauss' law and find the electric field in a distance r from an infinitely long (thin) rod with charge λ pr unit length.
Hint: Take advantage of the cylindrical symmetry of the system and find a useful gaussian surface.
(Compare your result with what you found in exercise 2 *d*) in øving 2.)

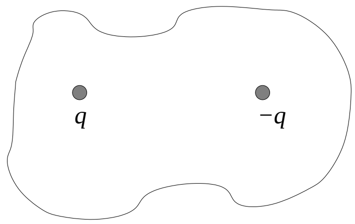
Exercise 4 (multiple choice)

a) On a closed surface, the electric field $\boldsymbol{E}$ is everywhere directed *inwards*. Then we may conclude that

A the surface normal $\hat{n}$ is parallel with $\boldsymbol{E}$ everywhere on the closed surface
B the surface encloses zero net charge
C the surface encloses a negative net charge
D the surface encloses a positive net charge

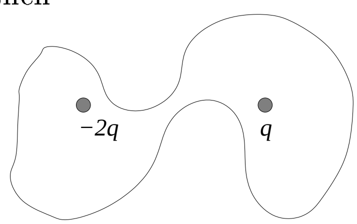
b) The figure illustrates a closed surface enclosing two point charges q og $-q$. The net electric flux out through this surface is then

A zero
B $-q/\varepsilon_0$
C q/ε_0
D $2q/\varepsilon_0$



c) The figure illustrates a closed surface enclosing two point charges $-2q$ og q. The net electric flux out through this surface is then

A zero
B $-q/\varepsilon_0$
C q/ε_0
D $2q/\varepsilon_0$



d) What is the radius of a (spherical) equipotential surface at 50 V with a point charge 10 nC in the centre? (Zero potential is chosen at infinity.)

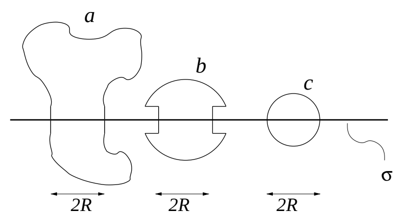
A 1.3 m
B 1.8 m
C 3.2 m
D 5.0 m

e) The potential in a region of space is $V(x, y, z) = 100$ V. The electric field $\boldsymbol{E}$ in this region is then

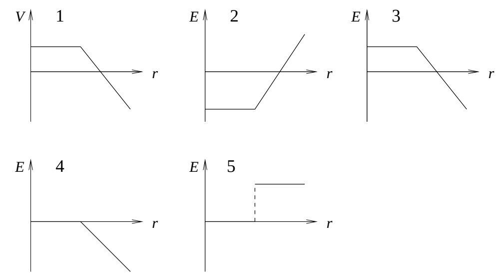
A (100 V/m) $\hat{x}$
B (100 V/m) $\hat{y}$
C (100 V/m) $\hat{z}$
D zero

f) A uniformly charged infinitely large surface has a charge σ pr unit area. Three gaussian surfaces (closed surfaces) a, b, and c are shown in the figure. All three surfaces enclose a circular disc with radius R when they cut through the charged surface. Range the three closed surfaces a, b, and c with respect to how much net electric flux that passes out through them.

A $a > b > c$
B $a > b = c$
C $a = b = c$
D $a < b < c$



g) If the potential V as a function of the distance r from a charge distribution is as given in graph nr 1, which graph then shows the electric field E as a function of the distance r?



A 2
B 3
C 4
D 5

h) The potential in a region is

$$V(x) = 50 \text{ V} + (15 \text{ V/m})x$$

The electric field in this region is then

A $50 \text{ V } \hat{x}$
B $(15 \text{ V/m})\ x\ \hat{x}$
C $(15 \text{ V/m})\ \hat{x}$
D $-(15 \text{ V/m})\ \hat{x}$

i) The potential in a region is

$$V(x, y, z) = (2 \text{ V/m})x + (3 \text{ V/m})y + (4 \text{ V/m})z$$

Then the x component of the electric field in this region is

A -2 V/m
B -3 V/m
C -4 V/m
D -9 V/m

j) A point charge q is located in one of the corners of a cube. What is the electric flux through the shaded side in the figure?

A q/ε_0
B $q/4\varepsilon_0$
C $q/8\varepsilon_0$
D $q/24\varepsilon_0$