Institutt for fysikk, NTNU
TFY4155/FY1003: Elektrisitet og magnetisme
Vår 2005

Øving 7

Guidance: February 24 and 25
To be delivered by: Monday February 28

Exercise 1

A sphere with radius R has spherically symmetric charge distribution (charge pr unit volume)

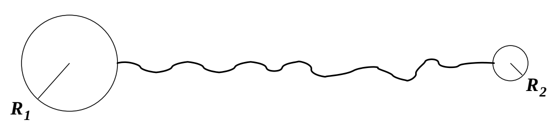
$$\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$$

for $r < R$. ($\rho = 0$ for $r > R$)

a) Sketch $\rho(r)$. What is the total charge on the sphere? [Answer: $8\pi\rho_0 R^3/15$]

b) Use Gauss' law and find $E(r)$. Sketch $E(r)$ between $r = 0$ and $r = 3R$.

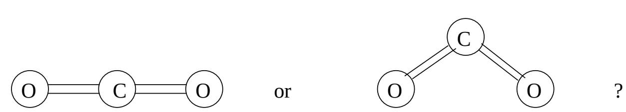
Exercise 2



Two metal spheres with radius R_1 and R_2, respectively, are connected via a long, thin metal wire. How will a net charge Q distribute itself on the two spheres? (We assume that the wire is so thin that we can neglect any net charge on it.) What is the electric field strength on the surface of each of the two spheres? (The distance between the two spheres is so large that we can neglect the direct interaction between them. Hence, both spheres have a spherically symmetric charge distribution on the surface.) Hint: First, find the potential on the spheres.

Exercise 3

Carbon dioxide, CO_2, has zero electric dipole moment. Does this imply that the molecule is *linear* or *bent*?



Ammonia, NH_3, has a nonzero permanent electric dipole moment whereas boron trifluoride, BF_3, has $p = 0$. Use this information to determine (roughly) what these two molecules look like.

Exercise 4

A parallel plate capacitor consists of two quadratic metal plates with side edges 50 cm. The distance between the plates is 1 cm.

a) What is the capacitance C_0 of this capacitor if we have air between the plates?

b) The two metal plates are given a certain amount of charge, Q and $-Q$, respectively. The potential difference between the plates is then found to be 96 V. What is the electric field between the plates? What is the plate charge Q?

c) The space between the plates is filled with water, resulting in a reduction of the potential between the plates. It is now found to be 1.2 V. What is then the relative premittivity ε_r, the electric susceptibility χ_e, and the permittivity $\varepsilon = \varepsilon_r \varepsilon_0$ of water?
(Note: We assume that the water is a perfect insulator. Hence, the potential drop is not caused by "leakage" of charge from one plate to the other. In reality, water always contains some ions, at least OH^- and H_3O^+. Therefore, in order to prevent charge transport between the capacitor plates, we could e.g. cover the plates with a thin layer of plastic before we added the water.)

d) Find the polariztion P in the water, i.e., the dipole moment pr unit volume. How much is this in comparison with the maximum theoretical polarization P_{max} in water? You will need this information: One water molecule has a dipole moment $6.2 \cdot 10^{-30}$ Cm. The molar volume of water is 18 cm^3. This means that in 18 cm^3 of water, there is one mole of water molecules, i.e., $6.02 \cdot 10^{23}$ molecules.

Some answers: *a*) 221 pF   *b*) $E = 9.6$ kV/m   *c*) $\varepsilon_r = 80$ *d*) $P/P_{\text{max}} \simeq 4 \cdot 10^{-7}$

Exercise 5

A sphere with radius R has uniform charge σ pr unit area on the surface of the "northern hemisphere" ($z > 0$) and uniform charge $-\sigma$ pr unit area on the surface of the "southern hemisphere" ($z < 0$). What is the dipole moment $\boldsymbol{p}$ of the sphere?   [Answer: $2\pi R^3 \sigma\, \hat{z}$]
Hint: First, find the dipole moment $d\boldsymbol{p}$ of two narrow rings, each with area $dA = (2\pi\rho) \cdot (R\, d\theta)$ and with charge $\pm dq = \pm\sigma\, dA$, positive on the ring on the northern hemisphere and negative on the ring on the southern hemisphere. The total dipole moment of the sphere is then determined by "summing" up such pairs of rings, i.e., by *integrating*, so that the whole surface has been accounted for.